Math 142, Exam 2, Fall 2016

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today. The quiz on Wednesday will be one problem from this exam.

No Calculators or Cell phones.

(1) (9 points) **Find** $\int \cos^5 x\, dx$**. Please check your answer.**

Let $u = \sin x$; so $du = \cos x dx$. Observe that

$$\int \cos^5 x\, dx = \int \cos^4 x \cos x\, dx = \int (1-\cos^2 x)^2 \cos x\, dx = \int (1-u^2)^2\, du$$

$$= \int (1 - 2u^2 + u^4) du = u - 2u^3/3 + u^5/5 + C$$

$$= \boxed{\sin x - (\frac{2}{3}) \sin^3 x + (1/5)\sin^5 x + C}.$$

<u>Check:</u> The derivative of the proposed answer is

$$\cos x - 2\sin^2 \cos x + \sin^4 x \cos x = \cos x(1 - 2\sin^2 x + \sin^4 x) = \cos x(1-\sin^2 x)^2$$

$$= \cos x(\cos^2 x)^2 = \cos^5 x. \checkmark$$

(2) (9 points) **Find** $\int \cos^4 x\, dx$**.**

Use $\cos^2 x = (1/2)(1+\cos 2x)$. Observe that

$$\int \cos^4 x\, dx = \int (\tfrac{1}{2}(1+\cos 2x))^2\, dx = \tfrac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx$$

$$= \tfrac{1}{4}\int (1 + 2\cos 2x + \tfrac{1}{2}(1+\cos 4x))\, dx$$

$$= \boxed{(\tfrac{1}{4})(x + \sin 2x + \tfrac{x}{2} + \tfrac{1}{8}\sin 4x) + C}.$$

(3) (8 points) **Find** $\displaystyle\int \frac{dx}{4x^2+8x+13}$**. Please check your answer.**

The integral is equal to

$$\int \frac{dx}{4(x^2+2x+\boxed{1})+13-4\boxed{1}} = \int \frac{dx}{(2(x+1))^2+9} = \frac{1}{9}\int \frac{dx}{((\frac{2}{3})(x+1))^2+1}.$$

Let $u = \frac{2}{3}(x+1)$; so $du = \frac{2}{3}dx$. The integral is equal to

$$\frac{1}{9}\frac{3}{2}\int \frac{du}{u^2+1} = \frac{1}{6}\arctan(u) + C = \boxed{\tfrac{1}{6}\arctan(\tfrac{2}{3}(x+1)) + C}.$$

Check: The derivative of the proposed answer is

$$\frac{1}{6}\frac{\frac{2}{3}}{[(\frac{2}{3}(x+1))^2+1]} = \frac{1}{9[\frac{4}{9}(x^2+2x+1)+1]} = \frac{1}{4x^2+8x+13}.\checkmark$$

(4) (8 points)**Find** $\displaystyle\int \frac{5x^3+3x+1}{(x^2+1)x^2}\,dx$**. Please check your answer.**

We first find numbers A, B, C, and D with

$$\frac{5x^3+3x+1}{(x^2+1)x^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x} + \frac{D}{x^2}.$$

Multiply both sides by $(x^2+1)x^2$ and equate the corresponding coefficients:

$$5x^3+3x+1 = (Ax+B)(x^2) + Cx(x^2+1) + D(x^2+1)$$

$$5x^3+3x+1 = Ax^3+Bx^2+Cx^3+Cx+Dx^2+D.$$

$$\begin{cases} 5 = A+C \\ 0 = B+D \\ 3 = C \\ 1 = D \end{cases}$$

So, $A = 2$, $B = -1$, $C = 3$, and $D = 1$. We verify that

$$\frac{5x^3+3x+1}{(x^2+1)x^2} = \frac{2x-1}{x^2+1} + \frac{3}{x} + \frac{1}{x^2}.$$

The right side is

$$\frac{(2x-1)x^2+3(x^2+1)x+(x^2+1)}{(x^2+1)x^2} = \frac{5x^3+3x+1}{(x^2+1)x^2}\checkmark$$

The integral is

$$\int \frac{2x-1}{x^2+1} + \frac{3}{x} + \frac{1}{x^2}dx = \boxed{\ln(x^2+1) - \arctan x + 3\ln|x| - \frac{1}{x} + C}.$$

(5) (8 points) **Find** $\int_2^{\infty} \frac{1}{x^2}\,dx$**.**

The integral is equal to

$$\lim_{b\to\infty} \int_2^b \frac{1}{x^2}\,dx = \lim_{b\to\infty} \left.\frac{-1}{x}\right|_2^b = \lim_{b\to\infty} \frac{-1}{b} + \frac{1}{2} = \boxed{\frac{1}{2}}.$$

(6) (8 points) **Consider the sequence described by** $a_1 = 1$**,** $a_2 = 1$**, and** $a_{n+2} = a_{n+1} + a_n$**, for** $1 \leq n$**. Write the first** 8 **terms of this sequence.**

$a_1 = 1$, $a_2 = 1$, $a_3 = 2$, $a_4 = 3$, $a_5 = 5$, $a_6 = 8$, $a_7 = 13$, $a_8 = 21$.