Math 142, Exam 1, Fall 2013

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Your work must be coherent, complete, and correct. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. **(9 points) Define the definite integral. Give a complete definition. Be sure to explain all of your notation. Write in complete sentences.**

Let $f(x)$ be a function defined on the closed interval $a \le x \le b$. For each partition P of the closed interval $[a,b]$ of the form $a = x_0 \le x_1 \le \cdots \le x_n = b$), let $\Delta_i = x_i - x_{i-1}$, and pick $x_i^* \in [x_{i-1}, x_i]$. The definite integral $\int_a^b f(x)dx$ is the limit over all partitions P as all Δ_i go to zero of $\sum\limits_{i=1}^{n} f(x_i^*)\Delta_i$.

2. **(9 points) Find $\int \sin^4 x \cos^5 x dx$. Check your answer.**

Let $u = \sin x$. In this case, $du = \cos x dx$ and

$$\int \sin^4 x \cos^5 x dx = \int \sin^4 x (1-\sin^2 x)^2 \cos x dx = \int u^4(1-2u^2+u^4)du$$

$$= \int (u^4 - 2u^6 + u^8)du = \frac{u^5}{5} - 2\frac{u^7}{7} + \frac{u^9}{9} + C = \boxed{\frac{\sin^5 x}{5} - 2\frac{\sin^7}{7} + \frac{\sin^9 x}{9} + C}$$

Check: The derivative of the proposed answer is

$$\sin^4 x \cos x - 2\sin^6 x \cos x + \sin^8 x \cos x = \sin^4 x \cos x (1 - 2\sin^2 x + \sin^4 x)$$

$$= \sin^4 x \cos x (1-\sin^2 x)^2 = \sin^4 x \cos^5 x. \checkmark$$

3. **(8 points) Find $\int \frac{dx}{4x^2+12x+13}$. Check your answer.**

Observe that $4x^2 + 12x + 13 = 4(x^2 + 3x + \boxed{\frac{9}{4}}) + 13 - 4\boxed{\frac{9}{4}} = 4(x+\frac{3}{2})^2 + 4 = (2x+3)^2 + 4$. It follows that

$$\int \frac{dx}{4x^2+12x+13} = \int \frac{dx}{(2x+3)^2+4} = \int \frac{dx}{4((x+\frac{3}{2}))^2+1)}$$

$$= \boxed{\tfrac{1}{4}\arctan(x+\tfrac{3}{2})+C}$$

Check: The derivative of the proposed answer is

$$\frac{1}{4((x+\frac{3}{2})^2+1)} = \frac{1}{4(x^2+3x+\frac{9}{4}+1)} = \frac{1}{4x^2+12x+13}.$$

4. **(8 points) Find** $\int \frac{dx}{x^2+x-12}$ **. Check your answer.**

Observe that $x^2+x-12=(x-3)(x+4)$. We use the technique of partial fractions:

$$\frac{1}{(x-3)(x+4)} = \frac{A}{x-3} + \frac{B}{(x+4)}.$$

$$1 = A(x+4) + B(x-3)$$

Plug in $x=-4$ to learn $B=\frac{-1}{7}$. Plug in $x=3$ to learn $A=\frac{1}{7}$. We check

$$\frac{1}{7}\left[\frac{1}{x-3} - \frac{1}{x+4}\right] = \frac{1}{7}\frac{7}{(x-3)(x+4)} = \frac{1}{x^2+x-12}.$$

So

$$\int \frac{dx}{x^2+x-12} = \frac{1}{7}\int \left[\frac{1}{x-3} - \frac{1}{x+4}\right] dx = \boxed{\tfrac{1}{7}\left[\ln|x-3| - \ln|x+4|\right]+C}$$

5. **(8 points) Find** $\int_{-2}^{4} \frac{dx}{(x-1)^2}$ **.**

The solution is on a separate piece of paper.

6. **(8 points) Find** $\int e^{2x}\cos(3x)dx$ **. Check your answer.**

Let $u=e^{2x}$ and $dv=\cos(3x)dx$. We calculate $du=2e^{2x}dx$ and $v=\frac{\sin(3x)}{3}$. We have

$$\int e^{2x}\cos(3x)dx = e^{2x}\frac{\sin(3x)}{3} - \frac{2}{3}\int e^{2x}\sin(3x)dx$$

Let $u=e^{2x}$ and $dv=\sin(3x)dx$. We calculate $du=2e^{2x}dx$ and $v=\frac{-\cos(3x)}{3}$.

$$\int e^{2x}\cos(3x)dx = e^{2x}\frac{\sin(3x)}{3} - \frac{2}{3}\int e^{2x}\sin(3x)dx$$

$$= e^{2x}\frac{\sin(3x)}{3} - \frac{2}{3}\left[e^{2x}\frac{-\cos(3x)}{3} + \frac{2}{3}\int e^{2x}\cos(3x)dx\right]$$

Add $\frac{4}{9}\int e^{2x}\cos(3x)dx$ to both sides.

$$\tfrac{13}{9}\int e^{2x}\cos(3x)dx = e^{2x}\tfrac{\sin(3x)}{3} + \tfrac{2}{9}e^{2x}\cos(3x)+C$$

$$\boxed{\int e^{2x}\cos(3x)dx = \tfrac{9}{13}\left[e^{2x}\tfrac{\sin(3x)}{3} + \tfrac{2}{9}e^{2x}\cos(3x)\right]+C}$$