Math 142, Exam 1, Fall 2016

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today. The quiz on Tuesday will be one problem from this exam.

No Calculators or Cell phones.

(1) **Find** $\int \cos^5 x\, dx$**. Please check your answer.**

Let $u = \sin x$; so $du = \cos x dx$. Observe that

$$\int \cos^5 x\, dx = \int \cos^4 x \cos x\, dx = \int (1-\cos^2 x)^2 \cos x\, dx = \int (1-u^2)^2\, du$$
$$= \int (1 - 2u^2 + u^4) du = u - 2u^3/3 + u^5/5 + C$$
$$= \boxed{\sin x - (2/3)\sin^3 x + (1/5)\sin^5 x + C}.$$

Check: The derivative of the proposed annswer is

$$\cos x - 2\sin^2 \cos x + \sin^4 x \cos x = \cos x(1 - 2\sin^2 x + \sin^4 x) = \cos x(1-\sin^2 x)^2$$
$$= \cos x(\cos^2 x)^2 = \cos^5 x. \checkmark$$

(2) **Find** $\int \cos^4 x\, dx$**.**

Use $\cos^2 x = (1/2)(1+\cos 2x)$. Observe that

$$\int \cos^4 x\, dx = \int (\tfrac{1}{2}(1+\cos 2x))^2\, dx = \tfrac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx$$
$$= \tfrac{1}{4}\int (1 + 2\cos 2x + \tfrac{1}{2}(1+\cos 4x))\, dx$$
$$= \boxed{(\tfrac{1}{4})(x + \sin 2x + \tfrac{x}{2} + \tfrac{1}{8}\sin 4x) + C}.$$

(3) **Find** $\int \frac{dz}{e^z + e^{-z}}$**. Please check your answer.**

Let $u = e^z$; so $du = e^z dz$. Observe that

$$\int \frac{dz}{e^z + e^{-z}} = \int \frac{e^z dz}{e^{2z}+1} = \int \frac{du}{u^2+1} = \arctan u + C = \boxed{\arctan e^z + C.}$$

Check: The derivative of the proposed answer is

$$\frac{e^z}{e^{2z}+1} = \frac{1}{e^z + e^{-z}}. \checkmark$$

(4) **Find** $\displaystyle\int e^{5x}\sin x\,dx$**. Please check your answer.**

Use integration by parts. Let $u = e^{5x}$ and $dv = \sin x dx$. Compute $du = 5e^{5x}dx$ and $v = -\cos x$. Observe that

$$\int e^{5x}\sin x\,dx = -e^{5x}\cos x + 5\int e^{5x}\cos x dx.$$

Use integration by parts again. Let $u = e^{5x}$ and $dv = \cos x dx$. Compute $du = 5e^{5x}dx$ and $v = \sin x$. Now we have

$$\int e^{5x}\sin x\,dx = -e^{5x}\cos x + 5\Big(e^{5x}\sin x - 5\int e^{5x}\sin x\,dx\Big).$$

Add $25\int e^{5x}\sin x\,dx$ to both sides to obtain

$$26\int e^{5x}\sin x\,dx = -e^{5x}\cos x + 5e^{5x}\sin x + C.$$

Conclude that

$$\int e^{5x}\sin x\,dx = (1/26)\big(-e^{5x}\cos x + 5e^{5x}\sin x\big) + K$$

$$= \boxed{(e^{5x}/26)\big(-\cos x + 5\sin x\big) + K.}$$

<u>Check:</u> The derivative of the proposed answer is

$$(e^{5x}/26)\big(\sin x + 5\cos x - 5\cos x + 25\sin x\big) = e^{5x}\sin x.\checkmark$$

(5) **Find the area of the region bounded by** $x + 4 = y^2$ **and** $y = x + 2$**. Please draw a meaningful picture.**

See the other page.