Quiz for September 20, 2005

Find \( \lim_{x \to 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2} \). Explain carefully which facts you are using.

**ANSWER:** We see that

\[
\lim_{x \to 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2} = \lim_{x \to 0} \left( \frac{3 \sin 3x^2}{\cos 3x^2} \cdot \frac{\sin 5x}{5x} + 25 \cdot \frac{\sin 5x}{5x} \right).
\]

We know that \( \lim_{t \to 0} \frac{\sin t}{t} = 1 \). We apply this fact with \( t \) replaced by \( 3x^2 \) and later with \( t \) replaced by \( 5x \). We know that the limit of a sum is the sum of the limits and the limit of a product is the product of the limits. The answer to our problem is

\[
3 \cdot 1 + 25 \cdot 1 \cdot 1 = 28.
\]