1. State both parts of the Fundamental Theorem of Calculus.

\[ f(x) \text{ be a continuous function.} \]

\[ a) \text{ If } A(x) = \int_{a}^{x} f(t) \, dt, \text{ then } A'(x) = f(x). \]

\[ b) \text{ If } F'(x) = f(x), \text{ then } \int_{a}^{b} f(x) \, dx = F(b) - F(a). \]

2. Let \( A(x) = \int_{1}^{x} t^{2} \, dt \). Find \( A'(x) \).

\[ A'(x) = \frac{1}{x} \]

3. Compute \( \frac{\pi}{2} \int_{0}^{\frac{\pi}{2}} \sin^{4} x \cos x \, dx \).

\[ \int_{0}^{\frac{\pi}{2}} \frac{\sin^{5} x}{5} \, dx = \frac{1}{5} \]
4. (10 points for each part) Consider the region $R$ which is bounded by $y = x^2$, $x = 2, x = 3$, and $y = 0$.
(a) Find the volume of $R$.
\[
\frac{3}{2} \int_2^3 x^2 \, dx = \frac{x^3}{3} \bigg|_2^3 = \frac{9}{3} - \frac{8}{3} = \frac{1}{3}
\]

(b) Find the volume of the solid which is obtained by revolving $R$ about the $x-$axis.
\[
\text{Vol} = \pi \int_2^3 x^4 \, dx = \pi \left[ \frac{x^5}{5} \right]_2^3 = \frac{\pi}{5} \left(3^5 - 32\right)
\]

(c) Find the volume of the solid which is obtained by revolving $R$ about the $y-$axis.
\[
\text{Vol} = 2\pi \int_2^3 x^3 \, dx = 2\pi \left[ \frac{x^4}{4} \right]_2^3 = \frac{\pi}{2} \left(3^4 - 16\right)
\]
5. Let \( f(x) = -x^3 + 3x^2 \). Where is \( f(x) \) increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of \( y = f(x) \). Graph \( y = f(x) \).

\[
\begin{align*}
  f'(x) &= -3x^2 + 6x \\
  &= -3x(x-2) \\
  f''(x) &= -6x \\
  &= -6(x-1) \\
  \frac{f''}{f'}(0) &= \frac{1}{2} \quad \text{f' neg} \\
  \frac{f''}{f'}(1) &= 0 \quad \text{f' pos} \\
  \frac{f''}{f'}(2) &= -1 \quad \text{f' neg}
\end{align*}
\]

\[\begin{array}{c}
\text{inc for } 0 < x < 1 \\
\text{dec for } x < 0 \text{ also } 2 < x \\
\text{cd for } x < 1 \\
\text{cd for } 1 < x \\
\text{loc. max } (1, 4) \\
\text{loc. min } (0, 0) \\
\text{pt of c } (1, 2)
\end{array}\]

6. Find the length of \( y = 2x^{3/2} \) between \( x = 1/3 \) and \( x = 7 \).

\[
\begin{align*}
\int_{\frac{1}{3}}^{7} \sqrt{1 + \left(\frac{3}{2} x^{1/2}\right)^2} \, dx &= \int_{\frac{1}{3}}^{7} \sqrt{1 + 9x} \, dx \\
&= \frac{2}{9} \left[ \frac{2}{3} x^{3/2} + 9x^{3/2} \right]_{\frac{1}{3}}^{7} \\
&= \frac{2}{27} \left( 64^{3/2} - 4^{3/2} \right) = \frac{2}{27} \left( 8^3 - 2^3 \right) \\
&= \frac{2 \cdot 8}{27} (64 - 1) = \frac{16}{27} (63)
\end{align*}
\]
7. Find the points on the curve \( y^2 + 2x = 9 \) which are closest to the point \((0, 0)\).

\[
\text{Minimize } Z = x^2 + y^2 \text{ on } y^2 = 9 - 2x
\]

so \( \text{minimize } Z = x^2 + 9 - 2x \)

\[
\frac{dZ}{dx} = 2x - 2
\]

\[
\frac{dx}{dx} = 0 \text{ when } x = 1
\]

**Ans** \((1, \sqrt{7})\) and \((1, -\sqrt{7})\)

8. Find the area of the surface which is generated by revolving \( y = x^{3/3} \), for \( 1 \leq x \leq \sqrt{7} \), about the \(x\)-axis.

\[
\text{Area} = \int_1^{\sqrt{7}} 2\pi x \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx
\]

\[
= \int_1^{\sqrt{7}} 2\pi x \sqrt{1 + (x^{3/3})^2} \, dx
\]

\[
= \frac{\pi}{9} \left( 50\pi^2 - 2\pi^2 \right)
\]