1. (12 points) Let $3x^3y^2 + \sin(2xy^2) = 4y^2 + 9x^2$. Find $\frac{dy}{dx}$.

$$3x^3 \frac{dy}{dx} + 9xy^2 + (x \frac{dy}{dx} + 2y^2) \cos(2xy^2) = 8y \frac{dy}{dx} + 18x$$

$$\frac{dy}{dx} = \frac{18x - 9xy^2 - 2y^2 \cos(2xy^2)}{6x^3 + 4xy \cos(2xy^2) - 8y}$$

2. (12 points) Let $y = \sqrt{\sin^4(9x^3 + 4x^2 + 19) + \cos^3 x}$. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{4 \sin^3 (9x^3 + 4x^2 + 19) \cos(9x^3 + 4x^2 + 19) (27x^2 + 8x) - 3 \cos^2 x \sin x}{2 \sqrt{\sin^4 (9x^3 + 4x^2 + 19) + \cos^3 x}}$$
3. (13 points) Find the dimensions of the open box of largest volume that can be made from a square piece of cardboard, 48 inches on each side, by cutting equal squares from the corners and turning up the sides.

\[ V = (48-2x)^2 x \quad 0 \leq x \leq 24 \]

\[ V' = 2(48-2x)(-2)x + \left(48-2x\right)^2 \]

\[ = (48-2x)\left[-4x + 48-2x\right] \]

\[ = \left[48-2x\right]\left[-6x + 48\right] \]

\[ = \left[48-2x\right]\left[6\right]\left(x-8\right) \]

The box should be 32 x 32 x 8''

4. (13 points) Each edge of a cube is growing at the constant rate of 5 inches per second. How fast is the surface area growing when each edge is 10 inches long?

\[ l = 10, \ \frac{dl}{dt} = 5 \]

\[ A = \text{Surface Area} \]

\[ A = 6l^2 \]

\[ \frac{dA}{dt} = 12l \frac{dl}{dt} \]

\[ \left. \frac{dA}{dt} \right|_{l=10} = 12(10)(5) = 600 \text{ in}^2 \text{/ sec} \]
5. (13 points) A rectangular box with a square base is to be constructed to hold 160 cubic yards of water. If the metal top costs five times as much per square yard as the concrete sides and base, what are the most economical dimensions for the box?

Let $l$ sq. yd. concrete cost $x$

Then $l$ sq. yd. metal costs $5x$

\[
C = \frac{4d}{\text{top}} x^2 + 4d \frac{\text{side}}{x^2} + \frac{\text{bottom}}{x^2}
\]

\[
C = \frac{5d}{x^2} x^2 + \frac{640d}{x} \quad 160 = x^2 y \quad \frac{160}{x} = y
\]

\[
C' = 10d x - \frac{640d}{x^2} \quad \text{the box should be 6 by 4 yds by 10 yds}
\]

\[
C' = 0 \quad \text{wke} \quad 10d \left[ x^2 - 64 \right] = 0
\]

\[
X = 4 \quad Y = 10
\]

6. (13 points) A student is using a straw to drink from a conical cup, whose axis is vertical, at the rate of 6 cubic inches per second. If the height of the cup is 10 inches and the radius of its opening is 7 inches, how fast is the level of the liquid falling when the depth of the liquid is 3 inches? (Recall that the volume of a cone is $V = \frac{1}{3} \pi r^2 h$.)

\[
V = \text{vol of the liquid}
\]

\[
l = \frac{3}{7} \quad \text{height of the liquid}
\]

\[
r = r = 7 - 3 \quad \text{radius of the liquid}
\]

\[
V = \frac{1}{3} \pi r^2 l = \frac{1}{3} \pi \frac{49}{100} 3^3
\]

\[
\frac{\text{dV}}{\text{dt}} = \pi \frac{49}{100} l^2 \frac{\text{dl}}{\text{dt}}
\]

\[
\frac{\text{dl}}{\text{dt}} = \frac{100}{49 \pi l^2} \frac{\text{dV}}{\text{dt}}
\]

\[
\frac{\text{dV}}{\text{dt}} = \frac{100}{49 \pi (9)} (-6) \text{ in/sec}
\]

\[
\frac{3}{7} \quad \text{in} \quad \frac{\text{dV}}{\text{dt}} = \frac{100}{49 \pi (9)} (-6) \text{ in/sec}
\]
7. (12 points) Let \( f(x) = -x^3 + 3x^2 - 2 \). Where is \( f(x) \) increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of \( y = f(x) \). Find all vertical and horizontal asymptotes. Graph \( y = f(x) \).

\[
\begin{align*}
f'(x) &= -3x^2 + 6x = -3x(x-2) \\
f''(x) &= -6x + 6 = -6(x-1)
\end{align*}
\]

No asymptotes.

\begin{align*}
f'(x) &= 0 \\
f'(x) &= 0 \\
f'(x) &= 0 \\
f'(x) &= 0
\end{align*}

\[
\begin{array}{c|c|c|c|c}
& f'(x) < 0 & f'(x) = 0 & f'(x) > 0 & f''(x) < 0 \\
\hline
0 & - & 0 & + & - \\
2 & + & 0 & - & + \\
\end{array}
\]

\[
\begin{align*}
f(0) &= 0 \\
f'(0) &= -2 \\
f(2) &= -8 + 12 - 2 = 2
\end{align*}
\]

\[
\begin{array}{c|c|c|c|c}
& f''(x) < 0 & f''(x) > 0 & f''(x) < 0 & f''(x) > 0 \\
\hline
-2 & - & + & - & + \\
2 & - & + & - & + \\
\end{array}
\]

\[
\begin{align*}
& 	ext{f is inc for } 0 \leq x \leq 2 \\
& 	ext{f is dec for } x < 0 \text{ and } 2 < x \\
& 	ext{f is c.u. for } x < 1 \\
& 	ext{f is c.d. for } 1 < x \\
& \text{local min at } (0, -2) \\
& \text{local max at } (2, 2) \\
& \text{pt. of inflection at } (1, 0) \\
& \text{no asymptotes}
\end{align*}
\]
8. (12 points) Let \( f(x) = \frac{1}{1 + x^2} \). Where is \( f(x) \) increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of \( y = f(x) \). Find all vertical and horizontal asymptotes. Graph \( y = f(x) \).

\[
\lim_{x \to \infty} f(x) = 0 \quad \lim_{x \to -\infty} f(x) = 0 \\
f'(x) = -\frac{2x}{(1 + x^2)^2} = -2x(1 + x^2)^{-2} \\
f''(x) = 4x(1 + x^2)^{-3} 2x - 2(1 + x^2)^{-2} \\
= \frac{2}{(1 + x^2)^3} \left[ 4x^2 - (1 + x^2) \right] \\
= \frac{2}{(1 + x^2)^3} \left[ 3x^2 - 1 \right]
\]

\[
\begin{align*}
\frac{f'(0)}{0} &\quad \frac{f'(\infty)}{0} \\
\frac{f''(-\infty)}{-\frac{1}{\sqrt{3}}} &\quad \frac{f''(\infty)}{\frac{1}{\sqrt{3}}} \\
\frac{f'' (-\frac{1}{\sqrt{3}})}{ } &\quad \frac{f'' (\frac{1}{\sqrt{3}})}{ }
\end{align*}
\]

Local max at \((0,1)\)

Local min at \((\frac{1}{\sqrt{3}}, \frac{2}{3})\) \((\frac{-1}{\sqrt{3}}, \frac{2}{3})\)

No vertical asymptotes.

\(y = 0\) is a hor. asy.

\(f\) is inc for \(x < 0\)

\(f\) is dec for \(x > 0\)

\(f\) is c.u. for \(x < -\frac{1}{\sqrt{3}}\)

\(f\) is c.d. for \(x \in (-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})\)

\(f\) is c.u. for \(x \in (\frac{1}{\sqrt{3}}, \infty)\)