1. (The penalty for each mistake is five points.) Let

\[ f(x) = \begin{cases} 
2 - x & \text{if } x < 0, \\
2 + x & \text{if } 0 \leq x \leq 1, \text{ and} \\
3 - x^2 & \text{if } 1 < x. 
\end{cases} \]

(a) Graph \( y = f(x) \).

(b) Fill in the blanks:

\[
\begin{align*}
 f(0) &= \frac{2}{\cancel{x}0} & \lim_{x \to 0^+} f(x) &= \frac{2}{\cancel{x}0} & \lim_{x \to 0^-} f(x) &= \frac{2}{\cancel{x}0} & \lim_{x \to 0} f(x) &= 2 \\
 f(1) &= \frac{3}{\cancel{x}1} & \lim_{x \to 1^+} f(x) &= \frac{3}{\cancel{x}1} & \lim_{x \to 1^-} f(x) &= \frac{3}{\cancel{x}1} & \lim_{x \to 1} f(x) &= \frac{3}{\cancel{x}1} \\
 f(2) &= -1 & \lim_{x \to 2^+} f(x) &= -1 & \lim_{x \to 2^-} f(x) &= -1 & \lim_{x \to 2} f(x) &= -1 
\end{align*}
\]

(c) Where is \( f(x) \) continuous?

Every where except \( x = 1 \).

(d) Where is \( f(x) \) differentiable?

Every where except \( x = 0 \) and \( x = 1 \).
2. Use the DEFINITION of the DERIVATIVE to find the derivative of 
\( f(x) = 4\sqrt{2x^3 - 3} \).
\[
\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{4\sqrt{(2(x+\Delta x)^3 - 3) - 4\sqrt{2x^3 - 3}}}{\Delta x}
\]
\[
= \lim_{\Delta x \to 0} \frac{4}{\Delta x} \left( \frac{\sqrt{2x^3 + 6\Delta x^3 - 3} - \sqrt{2x^3 - 3}}{\Delta x} \right)
\]
\[
= \lim_{\Delta x \to 0} \frac{4}{\Delta x} \left( \frac{\sqrt{2x^3 + 6\Delta x^3 - 3} - \sqrt{2x^3 - 3}}{\Delta x} \right) = \lim_{\Delta x \to 0} \frac{8x}{\Delta x} = \frac{8x}{\sqrt{2x^3 + 6\Delta x^3 - 3} + \sqrt{2x^3 - 3}}
\]
\[= \frac{8x}{2\sqrt{2x^3 - 3}} = \frac{4}{\sqrt{2x^3 - 3}}
\]

3. Find the equation of the line tangent to \( f(x) = x^5 - 3x^2 \) at \( x = 2 \).
\[f(2) = 32 - 12 = 20\]
\[f'(x) = 5x^4 - 6x\]
\[f'(2) = 80 - 12 = 68\]
\[y - 20 = 68(x - 2)\]

4. The position of an object above the surface of the earth is given by
\( s(t) = -16t^2 + 64t + 100 \), where \( s \) is measured in feet and \( t \) is measured in seconds. How high does the object get?
\[a' = -32 + 64\]
\[a' = 0 \text{ when } t = 2\]
\[\text{Max } E(t) = a(2) = -16(2) + 64(2) + 100 = 164 \text{ ft}\]
5. Let \( y = x^2 \cos^2(4x^5 + 19x) \). Find \( dy \).

\[
dy = \frac{dy}{dx} \, dx = \left[ x^2 \cdot 2 \cos(4x^5 + 19x) \right] \left[ \sin(4x^5 + 19x) \right] (20x^4 + 17) + 2x \cos^2(4x^5 + 19x) dx
\]

6. Let \( y = \sqrt{4x^3 + 9x + \sin^3(\cos(5x^4 + 3x))} \). Find \( \frac{dy}{dx} \).

\[
\frac{dy}{dx} = \frac{12x^2 + 9 \sin^2(\cos(5x^4 + 3x)) \cdot \cos(\cos(5x^4 + 3x)) \cdot \left[ \sin(5x^4 + 3x) \right] (20x^3 + 3)}{2 \sqrt{4x^3 + 9x + \sin^3(\cos(5x^4 + 3x))}}
\]
7. Let $3x^2y^3 = \sin(xy^2) + 3x^5$. Find $\frac{dy}{dx}$.

$$9x^2y^2 \frac{dy}{dx} + 6xy^3 = \cos(xy^2) \left[2xyy' + y^2 \right] + 15x^4$$

$$\frac{dy}{dx} \left[9x^2y^2 - 2xy\cos(xy^2) \right] = y^2 \cos(xy^2) + 15x^4 - 6xy^3$$

$$\frac{dy}{dx} = \frac{y^2 \cos(xy^2) + 15x^4 - 6xy^3}{9x^2y^2 - 2xy\cos(xy^2)}$$

8. Let $y = \frac{3}{x} + 15 - 4\sqrt{x}$. Find $\frac{d^2y}{dx^2}$.

$$y' = -3x^{-2} - 2x^{-\frac{3}{2}}$$

$$y'' = 6x^{-3} + x^{-\frac{3}{2}}$$
9. The area of a square is growing at the rate of 4 square feet per second. How fast is the length of each side growing when each side has length 6 feet?

Let \( A \) be the area of the square at time \( t \).

\[ A = x^2 \]

\[ \frac{dA}{dt} = 2x \frac{dx}{dt} \]

\[ \frac{dx}{dt} = \frac{1}{2x} \frac{dA}{dt} \]

\[ \frac{dx}{dt} \bigg|_{x=6} = \frac{4 \text{ ft}^2/\text{sec}}{12 \text{ ft}} = \frac{1}{3} \text{ ft/sec}. \]

10. A 30 foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the level pavement directly away from the wall at 3 feet per second, how fast is the top of the ladder moving down the wall when the foot of the ladder is 5 feet from the wall?

Let \( x \) be the distance from the base of the ladder to the wall.

Let \( y \) be the distance from the top of the ladder to the wall.

We know \( \frac{dx}{dt} = 3 \text{ ft/sec}. \) We want \( \frac{dy}{dt} \) when \( x = 5 \).

\[ 3^2 = x^2 + y^2 \]

\[ 0 = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \]

\[ -x \frac{dy}{dt} = \frac{dy}{dt} \]

\[ \frac{dy}{dt} \bigg|_{x=5} = \frac{-5}{\sqrt{900-25}} \cdot 3 \text{ ft/sec}. \]