EXTREMAL APPROXIMATELY CONVEX FUNCTIONS
AND THE BEST CONSTANTS IN A THEOREM OF
HYERS AND ULAM

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Abstract. Let $n \geq 1$ and $B \geq 2$. A real-valued function $f$ defined on the $n$-simplex $\Delta_n$ is approximately convex with respect to $\Delta_{B-1}$ if

$$f \left( \sum_{i=1}^{B} t_i x_i \right) \leq \sum_{i=1}^{B} t_i f(x_i) + 1$$

for all $x_1, \ldots, x_B \in \Delta_n$ and all $(t_1, \ldots, t_B) \in \Delta_{B-1}$. We determine the extremal function of this type which vanishes on the vertices of $\Delta_n$. We also prove a stability theorem of Hyers-Ulam type which yields as a special case the best constants in the Hyers-Ulam stability theorem for $\varepsilon$-convex functions.

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