THE COSMOLOGICAL TIME FUNCTION

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Abstract

Let \((M, g)\) be a time oriented Lorentzian manifold and \(d\) the Lorentzian distance on \(M\). The function \(\tau(q) := \sup_{p < q} d(p, q)\) is the \textbf{cosmological time function} of \(M\), where as usual \(p < q\) means that \(p\) is in the causal past of \(q\). This function is called \textbf{regular} iff \(\tau(q) < \infty\) for all \(q\) and also \(\tau \to 0\) along every past inextendible causal curve. If the cosmological time function \(\tau\) of a space time \((M, g)\) is regular it has several pleasant consequences: (1) It forces \((M, g)\) to be globally hyperbolic, (2) every point of \((M, g)\) can be connected to the initial singularity by a rest curve (i.e., a timelike geodesic ray that maximizes the distance to the singularity), (3) the function \(\tau\) is a time function in the usual sense, in particular (4) \(\tau\) is continuous, in fact locally Lipschitz and the second derivatives of \(\tau\) exist almost everywhere.

1991 \textit{Mathematics Subject Classification.} Primary: 53C50 Secondary: 83C75.

\textit{Key words and phrases.} Cosmological time function, globally hyperbolic, initial singularity.