CHARACTERIZATION OF EIGENFUNCTIONS
BY BOUNDEDNESS CONDITIONS

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Abstract

Suppose \( \{f_k(x)\}_{k=-\infty}^{\infty} \) is a sequence of functions on \( \mathbb{R}^n \) with \( \Delta f_k = f_{k+1} \) (where \( \Delta \) denotes the Laplacian) that satisfies the growth condition: \( |f_k(x)| \leq M_k (1+|x|)^a \) where \( a \geq 0 \) and the constants have sublinear growth \( \frac{M_k}{k} \to 0 \) as \( k \to \pm \infty \). Then \( \Delta f_0 = -f_0 \). This characterizes eigenfunctions \( f \) of \( \Delta \) with polynomial growth in terms of the size of the powers \( \Delta^k f, -\infty < k < \infty \). It also generalizes results of Roe (where \( a = 0, M_k = M, \) and \( n = 1 \)) and Strichartz (where \( a = 0, M_k = M, \) for \( n \)). The analogue holds for formally self-adjoint constant coefficient linear partial differential operators on \( \mathbb{R}^n \).

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