

SOLUTIONS NEAR SINGULAR POINTS TO THE EIKONAL AND RELATED FIRST ORDER NON-LINEAR PARTIAL DIFFERENTIAL EQUATIONS IN TWO INDEPENDENT VARIABLES

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ABSTRACT. A detailed study of solutions to the first order partial differential equation $H(x, y, z_x, z_y) = 0$, with special emphasis on the eikonal equation $z_x^2 + z_y^2 = h(x, y)$, is made near points where the equation becomes singular in the sense that $dH = 0$, in which case the method of characteristics does not apply. The main results are that there is a strong lack of uniqueness of solutions near such points and that solutions can be less regular than both the function H and the initial data of the problem, but that this loss of regularity only occurs along a pair of curves through the singular point. The main tools are symplectic geometry and the Sternberg normal form for Hamiltonian vector fields.

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