(1) Let $V$ be a vector space and $v_1, \ldots, v_m \in V$ with the property that if any vector is removed from \{v_1, \ldots, v_m\} then the resulting set has a smaller span than \{v_1, \ldots, v_m\}. Then $v_1, \ldots, v_m$ is linearly independent.

**Restatement:** If $S \subset V$ is a finite subset of $V$ such that for every proper subset $S' \subset S$ we have $\text{Span}(S') \neq \text{Span}(S)$, then $S$ is linearly independent.

**Hint:** One way would be to use the proposition on page 28 of the notes and a proof by contradiction.

(2) A vector space $V$ is **finite dimensional** if there it is spanned by a finite subset. (That is $V$ is finite dimensional if there are $v_1, \ldots, v_m \in V$ such that $V = \text{Span}\{v_1, \ldots, v_m\}$. Show that any finite dimensional vector space is spanned by a linearly independent set.