Mathematics 700 Homework
Due Wednesday, September 15

1. Find a basis for the set of vectors in $\mathbf{R}^4 = \{(x, y, z, w) : x, y, z, w \in \mathbf{R}\}$ so that the two conditions

$$\begin{aligned} x + 2y - 3z + w &= 0 \\ 2x - 3y + 5z - 2w &= 0 \end{aligned}$$

   hold.
2. Let V and W be subspaces of $\mathbf{R}^8$ so that $\dim V = 4$ and $\dim W = 5$. Show that there is a nonzero vector in $V \cap W$.
3. If V is a finite dimensional vector space and U a subspace of V then the ***codimension of*** W ***in*** V is defined by

$$\operatorname{codim}_V W := \dim V - \dim W.$$

   When the space V is clear from context we drop the subscript of V and just write $\operatorname{codim} W$. If U and W are subspaces of V then derive formulas for $\operatorname{codim}(U \cap W)$ and $\operatorname{codim}(U + W)$. (HINT: What to you know about $\dim(U \cap W)$ and $\dim(U + W)$?)
4. Let $\mathcal{P}_3$ be the vector space of all polynomials of degree ≤ 3 with real coefficients. Define two subsets $\mathcal{D}$ and $\mathcal{M}$ of $\mathcal{P}_3$ by

$$\begin{aligned} \mathcal{D} &:= \{p(x) : p(x+2) - 2p(x+1) + p(x) = 0\} \\ \mathcal{M} &:= \{p(x) : p(2x - 2) = 4p(x)\}. \end{aligned}$$

   Show that both $\mathcal{D}$ and $\mathcal{M}$ are subspaces of $\mathcal{P}_3$ and find bases for each of them.