

Mathematics 552

Quiz #1

Name: _____

In what follows U is an open subset of the complex plane \mathbf{C} and $z = x + iy$.

(1) Let $f(z)$ be a complex valued function defined on U . Define the derivative $f'(z)$ as a limit.

Solution:

$$f'(z) = \lim_{\Delta z \rightarrow 0} \frac{f(z + \Delta z) - f(z)}{\Delta z}.$$

(2) Let u be a real valued function defined on U . Define the partial derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ as limits.

Solution:

$$\frac{\partial u}{\partial x}(x, y) = \lim_{\Delta x \rightarrow 0} \frac{u(x + \Delta x, y) - u(x, y)}{\Delta x}$$

$$\frac{\partial u}{\partial y}(x, y) = \lim_{\Delta y \rightarrow 0} \frac{u(x, y + \Delta y) - u(x, y)}{\Delta y}$$

(3) State the Cauchy Riemannian equations.

Solution: If $f(z) = u(x, y) + iv(x, y)$ with u and v real valued and f is analytic in U then

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$