Homework assigned Friday, January 20.

We have seen that if a and r are complex numbers with $|r| < 1$ then the infinite series

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + \cdots = \frac{a}{1-r} = \frac{\text{first}}{1 - \text{ratio}}$$

Problem 1. Find the sum of the following series and draw a picture showing the set where the series converges:

(a) $5 + 5z^3 + 5z^6 + 5z^9 + \cdots$.

(b) $2 - 2 \cdot 3(z-4) + 2 \cdot 3^2(z-4)^2 - 2 \cdot 3^3(z-4)^3 + 2 \cdot 3^4(z-4)^4 + \cdots$.

(c) $\displaystyle\sum_{n=0}^{\infty} 2^n(z-5)^n$.

(d) $\displaystyle\sum_{n=3}^{\infty} 2^n(z-5)^n$.

Problem 2. In these problem, expand the given rational function into a series and draw a picture of where the series converges.

(a) $\dfrac{2}{1-z^4}$.

(b) $\dfrac{4}{1+3(z-4)}$.

(c) $\dfrac{z^4}{1-2(z+3)}$.