Homework assigned Wednesday, January 18.

Problem 1. Compute the following:

(a) The product $(z^2 + (3+i)z - 9)(2z^2 + 4iz - 9)$.
(b) The quotient and remainder when $z - 1 + 2i$ is divided into $z^3 - 3iz^2 + 4z + (2+3i)$.
(c) The roots of $z^2 + (2+2i)z + 1 + 2i = 0$. *Hint:* One way is to use the quadratic formula.

Problem 2. This problem shows that what you know about quadratic equations still holds with coefficients over the complex numbers. Let $p(z) = az^2 + bz + c$ where a, b, c are complex numbers and $a \neq 0$.

(a) Verify the identity

$$az^2 + bz + c = a\left(z + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

(b) Use (a) to show $az^2 + bz + c = 0$ is equivalent to

$$\left(z + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

(c) Use (b) to show the solutions to $az^2 + bz + c = 0$ are

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

We have seen every complex number has a square root, so this shows that the usual quadratic formula holds for roots of quadratic polynomials with complex coefficients.

Problem 3. Recall that we have shown that if $p(z)$ is a polynomial of degree n and that if α is a root of $p(z) = 0$, then $p(z)$ factors as $p(z) = (z - \alpha)q(z)$ where $q(z)$ has degree $(n-1)$. Use this and induction to show that a polynomial of degree n has at most n roots.