Homework assigned Friday, January 13.

Problem 1. Find the following.

(a) All the cube roots of i. Draw a picture of them.
(b) All the fourth roots of -16. Draw a picture of them.

Problem 2. Draw a picture showing all the fifth roots of $-4+4i$.

Problem 3. Let $p(z) = a_3 z^3 + a_2 z^2 + a_1 z + a_0$ where a_3, a_2, a_1, a_0 are *real* numbers. Show that if the complex number z_0 is a root of $p(z) = 0$, then the conjugate $\overline{z}_0$ is also a root of $p(z) = 0$. *Hint:* We know $p(z_0) = 0$. Take the complex conjugate of this equation and use that $\overline{a} = a$ for a real number.

Problem 4. Generalize the last problem to polynomials of arbitrary degree.

Problem 5. Use that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ and equate the real and imaginary part of both sides of the equations $e^{2i\theta} = (e^{i\theta})^2$ and $e^{3i\theta} = (e^{i\theta})^3$ to find formulas for $\cos(2\theta)$, $\sin(2\theta)$, $\cos(3\theta)$, and $\sin(3\theta)$.

Problem 6. Let θ_0 be a constant real number. Let $f\colon \mathbf{C} \to \mathbf{C}$ be the function $f(z) = e^{i\theta_0} z$. Explain why this is just rotation by an angle of θ_0 about the origin.