Mathematics 552 Homework due Friday, January 13

(1) Compute the following:

(a)
$$(1+3i)(4-2i)$$

(b) $\frac{3+2i}{4-i}$
(c) z^3 where $z = \frac{-1+\sqrt{3}i}{2}$.

- (2) Let z = 1 + i.
 (a) Find the polar form z = re^{iθ} of z.
 (b) Compute z¹⁵.
 (3) Let z = 1 − √3 i.
 - (a) Find the polar form $z = re^{i\theta}$ of z. (b) Compute z^{10} .
- (4) Show that for any integer *n* that $e^{2\pi ni} = 1$ and therefore $e^{(\theta + 2\pi n)i} = e^{\theta i}$.
- (5) Let n be a positive integer and k any integer. Let

$$\omega = e^{\frac{1}{n}(2k\pi i)}.$$

Show that

$$\omega^n = 1.$$

Thus ω is an *n*-th root of unity (i.e. an *n*-th root of 1.)