Mathematics 552 Test #1 Name:

Show your work! Answers that do not have a justification will receive no credit.

1. (30 Points) Compute the following:
   (a) \((4 - 3i)(6 + 2i)\)

   \[
   (b) \frac{5 + 2i}{3 + 4i}
   \]

   (c) \(\text{Arg}(-3 - 3i)\)

   (d) \(\text{arg}(-1 + \sqrt{3}i)\)

   (e) \((-1 + \sqrt{3}i)^{11}\)

   (f) \[
   \left| \frac{(\sqrt{3} + i)^9}{(\sqrt{3} - i)^8} \right|
   \]

   (g) \(e^{2 + \frac{\pi}{4}i}\)

   (h) \(\text{Im}[(4 - 5i)(x + yi)]\)
2. (10 Points) Find all values of \((-27)^{\frac{2}{3}}\). 

3. (10 Points) Show that \(e^z + e^{\bar{z}} = 2e^{\text{Re}z} \cos(\text{Im} z)\)
4. (15 Points) Draw pictures of the following sets of complex numbers:
(a) \(|z - 4 - 3i| < 2\)

(b) \(2 < |z| < 3\) and \(\frac{\pi}{2} < \text{Arg}(z) < \pi\)

(c) \(\text{Re}[(2 - 3i)z] < 6\)
5. (10 Points) Let
\[ A = \{ z : 1 < |z| < 3 \}, \quad B = \{ z : |z| \geq 5 \}, \quad C = \{ z : \text{Re} z > 0, |z| \leq 4 \} \]
Then
(a) Which of these sets is open? __________________________

(b) Which of these sets are domains? _________________________

(c) Which of these sets are regions? ________________________

(d) Which of these sets are bounded? ________________________

6. (15 Points) Solve the following equations:
(a) \[ \frac{1 + z}{1 - z} = 3 + 4i \]
\[ z = \] __________________________

(b) \[ z^2 + (-3 + 3i)z - 5i = 0 \]
\[ z = \] __________________________
7. (10 Points) One cube root of $-512$ is $4 + 4\sqrt{3}i$. Plot all of the cube roots.