

# Test 3

Name: \_\_\_\_\_

**Show your work!** Answers that do not have a justification will receive no credit.

1. (40 points) Compute the following:

(a)  $\int_{\gamma} (3 - 2\bar{z}) dz$  where  $\Gamma$  is the curve parameterized by  $z(t) = 2t + ti$ ,  $0 \leq t \leq 2$ .

(b)  $\int_{\Gamma} (6z^2 - 4z + 2) dz$  where  $\Gamma$  is the curve  $z(t) = t^2 e^{(1-t)^2 i}$ .

(c)  $\int_{|z|=3} \frac{e^z \cos(z) dz}{(z-1)(z+7)}$

(d)  $\int_{|z|=2} \frac{dz}{(z-1)^2}$

2. (25 points) Let  $D = \{z : 1 < |z| < 3\}$

(a) Sketch the graph of  $D$ .

(b) Compute  $\int_{|z|=2} \frac{dz}{z}$ .

(c) The function  $f(z) = \frac{1}{z}$  is analytic in  $D$ . Prove that  $f(z)$  does not have an anti-derivative in  $D$ .

3. (15 points) (a) What is the domain of analyticity of  $f(z) = \frac{\cos(z)}{z^2 + 9}$ ?

(b) For this function what is the radius of convergence when it is expanded as a power series about the point  $z = 4$ ?

4. (10 points) Explain why  $\cos(z^2)$  has an anti-derivative on  $\mathbb{C}$ .

5. (10 points) Let  $f(z) = u + iv$  be an entire function so that  $|u| \leq 3$  and  $|v| \leq 4$ . Then what can you say about  $f(z)$ ?