1. (30 points) Complete the following identities:

(a) $\nabla(fg) =$

(b) $\text{div}(f\mathbf{F}) =$

(c) $\text{div}(\mathbf{F} \times \mathbf{G}) =$

(d) $\text{div} \text{ curl} \mathbf{F} =$

(e) $\text{curl}(f\mathbf{F}) =$

(f) $\frac{d}{dt}(\mathbf{b}(t) \times \mathbf{c}(t)) =$

2. (10 points) What are the velocity and acceleration of the path $\mathbf{c}(t) = (t, t^2, t^3)$?

Velocity = __________________________

Acceleration = __________________________
3. (10 points) Sketch the graph of the curve parameterized by $x(t) = 3 \cos(t)$ and $y(t) = 2 \sin(t)$.

4. (15 points) Let $f(x, y) = x^2 - xy + y^3$.
   (a) What the equation of the tangent to $z = f(x, y)$ at the point $(1, 2, 7)$?

   (b) Where does the tangent plane intersect the $z$-axis?
5. (10 points) What is the tangent line to \( c(t) = (t^2, t^3) \) when \( t = 2 \)?

6. (5 points) Let \( f = xy + yz + xz \). Then compute the gradient of \( f \).

\[
\nabla f =
\]

7. (10 points) Let \( \mathbf{F} = yzi + xzj + kxy^2 \). Then compute \( \text{curl} \, \mathbf{F} \).

\[
\text{curl} \, \mathbf{F} =
\]
8. (10 points)
(a) Let $V : \mathbb{R}^3 \to \mathbb{R}$ be a function and $c : [a, b] \to \mathbb{R}^3$ a path. Then state the chain rule for
\[
\frac{d}{dt} V(c(t)) =
\]

(b) Now assume that $c(t)$ satisfies
\[
mc''(t) = -\nabla V(c(t))
\]
for a positive number $m$ (the “mass”). The show that
\[
E = \frac{1}{2}m\|c(t)\|^2 + V(c'(t))
\]
is constant.