Mathematics 546 Test #1 Name: ____________________________

Show your work! Answers that do not have a justification will receive no credit.

(1) (25 Points) Define the following:
   (a) $a$ is a factor of $b$.

   (b) $p$ is a prime.

   (c) $d$ is the greatest common divisor of $a$ and $b$.

   (d) The the subset $I$ of $\mathbb{Z}$ is an ideal.

   (e) $(G, \cdot)$ is a group.
(2) (20 Points) State the following:
   (a) The principle of induction.

   (b) The Trichotomy principle.

   (c) The division algorithm.

   (d) The “gcd= linear combo” theorem.

(3) (10 Points) Find a solution to $4x \equiv 5 \mod 13$. 
(4) (10 Points) Find the greatest common divisor of 153 and 180 and express gcd(153, 180) as a linear combination of 153 and 180.

(5) (10 Points) Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.

(6) (10 Points) Give the addition and multiplication tables for arithmetic modulo 5.
(7) (10 Points) Prove that if \( a \equiv b \mod m \), then for any integer \( s \) that \( as \equiv bs \mod m \).

(8) (10 Points) Show that for any integer \( n \) that \( \gcd(n, n + 3) \) is either 1 or 3.