Mathematics 532/

Quiz # 1 Name:________________________________

***Y*ou must show your work to get full credit.**

For this quiz $\mathbb{A}^2$ will be the affine space of ordered pairs (x, y) with $x, y \in \mathbf{R}$ where $\mathbf{R}$ is the set of real numbers.

1. For $a, b, c \in \mathbf{R}$ define $L(a, b, c)$.

This is the set of points

$$L(a, b, c) = \{(x, y) \in \mathbb{A}^2 : ax + by + c = 0\}$$

where a and b are not both zero. $\square$

2. Let A and B be points of $\mathbb{A}^2$. Then define P is an ***affine combination*** of A and B.

This means

$$P = \alpha A + \beta B$$

where $\alpha, \beta \in \mathbf{R}$ and

$$\alpha + \beta = 1.$$

$\square$

3. Let A, B, and C be points of $\mathbb{A}^2$. Then define P is an ***affine combination*** of A, B and C.

This means

$$P = \alpha A + \beta B + \gamma C$$

where $\alpha, \beta, \gamma \in \mathbf{R}$ and

$$\alpha + \beta + \gamma = 1.$$

$\square$

4. Show that if A and B are one the line $L(a, b, c)$ then so is any affine combination of A and B.

Proof. Let $A = (x_1, y_1)$ and $B = (x_2, y_2)$. We are assuming that A and B are on $L(a, b, c)$ which means that

$$ax_1 + by_1 + c = 0 \qquad \text{and} \qquad ax_2 + by_2 + c = 0. \tag{1}$$

As P is an affine combination of A and B we have that there are $\alpha, \beta \in \mathbf{R}$ with $\alpha + \beta = 1$ and

$$P = \alpha A + \beta B = \alpha(x_1, y_1) + \beta(x_2, y_2) = (\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2).$$

To finish the proof, we just need to check that the coordinates of P satisfy the equation defining $L(a, b, c)$.

$$\begin{aligned}
a(\alpha x_1 + \beta x_2) + b(\alpha y_1 + \beta y_2) + c &= \alpha(ax_1 + by_1) + \beta(ax_2 + by_2) + c \\
&= \alpha(-c) + \beta(-c) + c && \text{(by the equations 1)} \\
&= -c(\alpha + \beta) + c \\
&= -c(1) + c && (\text{as } \alpha + \beta = 1) \\
&= 0
\end{aligned}$$

Therefore P is on $L(a, b, c)$. $\square$