**Mathematics 527 Test #2** Name:____________________________

**Show your work to get credit.** An answer with no work will not get credit.

(1) (5 points) State the $n$-th order Taylor theorem about $x$ and with remainder for $f(x+h)$.

(2) (5 points) Let $f$ be a function on $[a,b]$ and $x_0,\ldots,x_n$ distinct points of $[a,b]$. Then what does it mean for the polynomial $p(x)$ to interpolate $f$ at the points $x_0,\ldots,x_n$?

(3) (5 points) Let $f$ be $n+1$ times differentiable on $[a,b]$ and let $p(x)$ be the polynomial of degree $\le n$ that interpolates $f$ at the distinct points $x_0,x_1,\ldots,x_n \in [a,b]$. What is the formula for the error $f(x)-p(x)$?

(4) (10 points) Let $x_0,\ldots,x_n$ be distinct points of $\mathbf{R}$.

(a) Define the ***cardinal functions*** $\ell_0,\ldots,\ell_n$ determined by these points.

(b) If $n \ge 2$ explain why $\displaystyle\sum_{i=0}^{n} x_i^2 \ell_i(x) = x^2$.

(5) (15 points) Construct Newton's interpolating polynomial for the data (you do not have to simplify your answer)

| $x$ | -1 | 1 | 3 | 4 |
|---|---|---|---|---|
| $y$ | -9 | 2 | -3 | -4 |

(6) (20 points) Complete the following table of divided differences.

| $x$ | $f[\,]$ | $f[\,,]$ | $f[\,,\,,]$ | $f[\,,\,,\,,]$ |
|---|---|---|---|---|
| 1 | $-1$ | | | |
| 3 | 5 | | | |
| 5 | 11 | | | |
| 6 | 59 | | | |

(7) (20 points) A interpolating polynomial of degree 20 is used to approximate $\sin(x)$ on the interval $[-1, 1]$ at 21 equally spaced nodes. How accurate will this be?

(8) (20 points) Determine the error term in the approximation

$$f'(x) \approx \frac{1}{2h}[4f(x+h) - 3f(x) - f(x+2h)]$$