Mathematics 527 Test 3 Name: _____

You will need a calculator. You may use one page (both sides) of notes during the test.

1. (25 points.) Let

$$\int_0^3 f(x)\,dx \approx w_0 f(x_0) + w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3)$$

be the Gaussian quadrature method using four nodes in the interval $[0, 3]$.

(a) Explain carefully what this means. That is how are the nodes and weights chosen and for what class of functions is the formula exact.

(b) What is the sum $w_0 + w_1 + w_2 + w_3$? Prove your answer is correct using the properties Gaussian quadrature.

2. (10 points) Let $f(x, y)$ be a function so that

$$f(1,2) = 1, \quad f_x(1,2) = 2, \quad f_y(1,2) = 3,$$
$$f_{xx}(1,2) = 4, \quad f_{xy}(1,2) = 5, \quad f_{yy}(1,2) = 6.$$

Then give an approximation to $f(1.2, 1.9)$ using Taylor's theorem.

3. (15 points) Let $x(t)$ satisfy the differential equation $\dot{x}(t) = f(t, x(t))$. The give formula's for $\ddot{x}(t)$ and $\dddot{x}(t)$ that only involve $f(t, x)$ and its partial derivatives.

$\ddot{x}(t) =$

$\dddot{x}(t) =$

4. (20 points)

(a) Apply 2 steps of the four order Runge-Kutta method to the initial value problem

$$\dot{x}(t) = f(t), \quad x(0) = 0$$

to give a formula for an approximation to $x(1)$.

(b) Explain why this gives an approximation to $\int_0^1 f(s)\, ds$.