Mathematics 527 Test 2 Name:____________________

You will need a calculator. You may use one page (both sides) of notes during the test.

1. (25 points.) A function $f(x)$ is only known by the table of values:

x	1	3	4
$f(x)$	2	1	5

   (a) What is the polynomial $p(x)$ of degree ≤ 2 that interpolates this data.

   (b) Give an estimate of $f(1.5)$ and give a brief explanation of why you believe this estimate is reasonable.

   (c) Give an estimate for the derivative $f'(2)$.

(d) Approximate $\int_1^4 f(x)\, dx$ by

i. The trapezoid rule

ii. and $\int_1^4 p(x)\, dx$.

(e) Do you have an opinion as to which of these approximations to $\int_1^4 f(x)\, dx$ might give the best estimate to the correct value?

2. (15 points.) Let $p_n(x)$ be the polynomial that interpolates the function $f(x) = e^{-x}$ on the interval $[0, 2]$ at $n + 1$ equally spaced nodes. Then how large to we have to take n so that the error in approximating e^{-x} by $p_n(x)$ is $\leq 10^{-6}$?

3. (15 points.) Let $f(x)$ be a function with derivatives of all orders. Let

$$\varphi(h) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

(a) Let $\varphi(h) = f''(x) + E(h)$ where $E(h)$ is the error term. Use Taylor's theorem to derive a formula for the error term $E(h)$.

(b) Assuming that $\varphi(h)$ can be expanded as

$$\varphi(x) = f''(x) + a_2(x)h^2 + a_4(x)h^4 + a_6(x)h^6 + \cdots$$

then find an approximation to $f''(x)$ that is of order $O(h^4)$.

4. (15 points.) How large to you have to take n in the trapezoid rule to compute the integral $\int_0^3 e^{-x^2}\, dx$ to five decimal places?

5. (10 points.) Let $T(n)$ be the trapezoid sum for the integral $\int_a^b f(x)\,dx$ on the interval $[a,b]$ for a partition $\mathcal{P} = \{a = x_0, x_1, \ldots, x_{2^n} = b\}$ of 2^n equally spaced intervals. Derive the recursive formula:

$$T(n) = \frac{1}{2}T(n-1) + h\sum_{k=0}^{2^{n-1}-1} f(x_{2k+1})$$

where $h = \dfrac{b-a}{2}$.

6. (15 Points) Approximate $\int_0^{.5} \frac{\sin(x)}{x}\, dx$ to five decimal places by use of Taylor's theorem.