## Mathematics 527 Test 1 Name:

1. (15 points) Find the Taylor expansion for the following functions about the indicated points.
(a) $e^{-3 x}$ about $x=0$
(b) $\sin (2)$ about the point $x=\frac{\pi}{4}$
(c) $F(x)=\int_{0}^{x} \cos \left(t^{2}\right) d t$
2. (10 points) Using your answer to part (c) of the last question compute $\int_{0}^{x} \cos \left(t^{3}\right) d t$ accurate to five decimal places and explain why you believe that your answer is correct.
3. (10 points) Let $H(t)$ be a function so that $H(1)=.5, H^{\prime}(1)=-2$, and $H^{\prime \prime}(1)=.2$.
(a) Give the best estimate you can for $H$ (1.1) and explain briefly why you think your estimate is good.
(b) Draw a graph of $y=H(t)$ near $t=1$.
4. (15 points) Let $f$ be a continous function so that $f(1)=-2$ and $f(3)=1$. How many steps of the bisection method does it require to get compute a root of $f(x)=0$ accurate to five decimal places?
5. Consider the function $y=f(x)$ with graph

(a) What are the roots of $f(x)$ between 0 and 8 ?
(b) For the following graph show what the first three steps in Newton's method starting at $x_{0}=6.6$.
(c) Starting at $x_{0}=6.6$ to which of the roots does Newton's method converge?
6. (10 points) Again consider the function $y=f(x)$ with graph

(a) Starting at $x_{0}=7$ and $x_{1}=3$ show the first three steps in the secant method.
(b) Starting at $x_{0}=7$ and $x_{1}=3$ to which root does the secant method converge?
7. (10 points)
(a) Draw a graph of a function and a starting position where Newton's method fails.
(b) Draw a graph of a function and starting points $x_{0}$ and $x_{1}$ where the secant method fails.
