1. (15 points) Find the Taylor expansion for the following functions about the indicated points.

(a) $e^{-3x}$ about $x = 0$

(b) $\sin(2)$ about the point $x = \frac{\pi}{4}$

(c) $F(x) = \int_0^x \cos(t^2) \, dt$
2. (10 points) Using your answer to part (c) of the last question compute \( \int_0^x \cos(t^3) \, dt \) accurate to five decimal places and explain why you believe that your answer is correct.
3. (10 points) Let $H(t)$ be a function so that $H(1) = .5$, $H'(1) = -2$, and $H''(1) = .2$.

(a) Give the best estimate you can for $H(1.1)$ and explain briefly why you think your estimate is good.

(b) Draw a graph of $y = H(t)$ near $t = 1$.

4. (15 points) Let $f$ be a continuous function so that $f(1) = -2$ and $f(3) = 1$. How many steps of the bisection method does it require to get compute a root of $f(x) = 0$ accurate to five decimal places?
5. Consider the function \( y = f(x) \) with graph

(a) What are the roots of \( f(x) \) between 0 and 8?

(b) For the following graph show what the first three steps in Newton’s method starting at \( x_0 = 6.6 \).

(c) Starting at \( x_0 = 6.6 \) to which of the roots does Newton’s method converge?
6. (10 points) Again consider the function \( y = f(x) \) with graph

(a) Starting at \( x_0 = 7 \) and \( x_1 = 3 \) show the first three steps in the secant method.

(b) Starting at \( x_0 = 7 \) and \( x_1 = 3 \) to which root does the secant method converge?
7. (10 points)

(a) Draw a graph of a function and a starting position where Newton’s method fails.

(b) Draw a graph of a function and starting points $x_0$ and $x_1$ where the secant method fails.