Math/Stat 511 Final

Show your work! Answers that do not have a justification will receive no credit.

1. (15 Points)
   (a) Define what it means for events $A_1, \ldots, A_k$ to be mutually exclusive.

   (b) Define what it means for events $A$, $B$, and $C$ to be independent.

   (c) Let $A$ and $B$ be events with $P(B) \neq 0$. Then what is the definition of the conditional probability $P(A|B)$?

   (d) Complete the following: **Probability** is a set function that assigns to each event $A$ in the sample space $S$ a number $P(A)$, called the probability of the event $A$, such that the following properties are satisfied:

   (e) Let $X$ be a discrete random variable with space $R$. Then define the probability density function (p.d.f.) of $X$.

   (f) If $X$ is a discrete random variable with space $R$ and p.d.f. $f(x)$ then define the mathematical expectation $E(u(X))$ of $u(X)$. 
2. (5 Points) If \( S = B_1 \cup B_2 \) with \( B_1 \cap B_2 = \varnothing \) and \( A \) is an event with \( P(A) \neq 0 \) then give the derivation of Bayes’ Law for \( P(B_2|A) \). (It is fine if you do this in terms of a tree diagram.)

3. (10 Points) Let \( A \) and \( B \) be events so that \( P(A) = .5 \), \( P(B) = .7 \), \( P(A \cup B) = .8 \). Then compute
   (a) \( P(A') \)
   \[ P(A') = \] 
   (b) \( P(A \cap B) \)
   \[ P(A \cap B) = \] 
   (c) \( P(A \cup B') \)
   \[ P(A \cup B') = \]

4. (5 Points) Assume the events \( A \) and \( B \) are independent and that \( P(A) = .5 \) and \( P(B) = .2 \). Then what is \( P(A \cup B) \)?
   \[ P(A \cup B) = \]
5. (5 Points) Alice and Bill play a checker tournament where the first person to win 5 games wins the tournament. How many different ways can Alice win the tournament in exactly 8 games?

6. (10 Points) Let $A_1$ and $A_2$ be the events that that a person is left or right handed, respectively. Let $B_1$ and $B_2$ be the events that a person is left eye dominant or right eye dominant, respectively. A survey in one statistics class yielded the following data:

<table>
<thead>
<tr>
<th></th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>20</td>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td>$A_2$</td>
<td>30</td>
<td>25</td>
<td>55</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td>40</td>
<td>90</td>
</tr>
</tbody>
</table>

Compute the following:
(a) $P(A_2)$

(b) $P(B_2|A_1)$

(c) The probability that a member of the class is left handed given that they are right eye dominant.
7. (10 Points) Assuming that it is equally likely that a person be born on any of the 7 days of the week, then what is the probability that of 7 people chosen at random no two were born in the same day of the week?

___________________________________________

8. (10 Points) Two cards are drawn at random without replacement from a standard deck of 52 playing cards. Then compute
(a) The probability that one is a club and the other is a heart.  ____________________________

(b) The first one is a King and the second one is Black. ____________________________

9. (10 Points) Students coming form high school district A have a 70% chance of passing freshman calculus, while students coming from high school district B have a 80% chance of passing the class. If a freshman calculus class has 25% of its students from district A and the remaining 75% from district B, then what is the probability that a student who passes the class can from district A?

___________________________________________
10. (10 Points) Let $X$ be a discrete random variable with p.d.f.

$$
f(x) = \frac{3 + x}{6}, \quad x = -2, -1, 0.
$$

Find the mean and variance of $X$.

$$
\mu = \underline{\underline{\underline{\text{ }}}}
$$

$$
\sigma^2 = \underline{\underline{\underline{\text{ }}}}
$$

11. (5 Points) A box of 12 donuts has 8 plain and 4 chocolate donuts. If 6 donuts are chosen at random then what is the probability that exactly 2 are chocoate?

\underline{\underline{\underline{\text{ }}}}

12. (10 Points) If 20% of the students at U.S.C. are left handed, let $X$ be the number of left handed people out of a random sample of 20 students.

(a) What is the expected number of people in the sample that are left handed. \underline{\underline{\underline{\text{ }}}}

(b) Compute $P(3 \leq X \leq 6)$ \hspace{1cm} $P(3 \leq X \leq 6) = \underline{\underline{\underline{\text{ }}}}$
13. (20 points) Let \( X \) be a random variable with the given function \( M(t) \) as moment generating function. Then fill in the required information about \( X \).

(a) \( M(t) = (.7 + .3e^t)^5 \)
   (i) What is the distribution of \( X \)?

(ii) What is the pdf of \( X \)?

(iii) What is \( E(X) \)? \( E(X) = \) ________

(iv) What is \( P(X = 3) \)? \( P(X = 3) = \) ________

(b) \( M(t) = e^{-2t+24t^2} \)
   (i) What is the distribution of \( X \)?

(ii) What is the pdf of \( X \)?

(iii) What is the expect value of \( X \)? \( \mu = \) ________

(iv) What is the variance of \( X \)? \( \sigma^2 = \) ________

(c) \( M(t) = .1e^{-2t} + .7e^t + .2e^{3t} \).
   (i) What is the pdf of \( X \)?

(ii) What is \( P(X = 6) \)? \( P(X = 6) = \) ________
14. (10 points) Let $X$ be a random variable of continuous type with pdf

$$f(x) = \begin{cases} \quad cx(1-x) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of $c = \underline{\phantom{1234567890}}$

(b) What is the expect value of $X$? $E(X) = \underline{\phantom{1234567890}}$

(c) What is the distribution function of $X$?

15. (5 points) Let $X$ have a normal distribution with mean $\mu = 3$ and variance $\sigma^2 = 4$. Then find the following probabilities. Then find the probability $P(1 \leq X \leq 7)$

$$P(1 \leq X \leq 7) = \underline{\phantom{1234567890}}$$

16. (10 Points) Let $X$ have the p.d.f. $f(x) = 5x^4$ for $0 \leq x \leq 1$. Then find the p.d.f. of $Y = X^{1/3}$. 