Show your work! Answers that do not have a justification will receive no credit.

1. (30 points) Find a particular solution to each of the following:
   (a) \[ y'' - 8y' + 25y = 4\cos(3x) \]
   (b) \[ y'' + 4y' - 8y = 3x^2 - 4x + 9 \]
   (c) \[ y'' - y' - 2y = 3 + 4e^{2x} \]
2. (15 points) The functions \( y_1 = x \) and \( y_2 = x^2 \) are solutions to 
\[ x^2 y'' - 2y' + 2y = 0. \] 
Find the general solution to \( x^2 y'' - 2y' + 2y = \frac{3}{x} \)
3. (15 points) The function $y_1 = x$ is a solution to $5x^2y'' + xy' - y = 0$.
Find the general solution to this equation.
4. (10 points) Let \( L[y] = y'' + py' + qy \). Then show that \( L[c_1y_1 + c_2y_2] = c_1L[y_1] + c_2L[y_2] \).

5. (15 points) A mass of weighting 12 lbs is attached both a vertically suspended spring with a spring constant of \( k = 6 \) and a dashpot that provides 3 lbs of resistance for every foot per second of velocity (that is the dashpot constant is \( c = 3 \)). If the mass is pulled down 1 ft below its static equilibrium position and released from rest,

(a) Find a formula for the position of the mass after \( t \) seconds. (Use \( g = 32\text{ft/sec}^2 \)).
6. (15 points). A mass weighing 100 lb is attached to the end of a spring with spring constant \( k = .01 \). A force of \( F = F_0 \cos(\omega t) \) acts on the mass. At what value of \( \omega \) will resonance occur? (That is what is the value of \( \omega \) leads to unbounded solutions?) (Note it may be possible to answer this question without find the complete solution to the equation.)