Mathematics 174 Test #2

Name:

Show your work to get credit. An answer with no work will not get credit.

1. (25 Points) Evaluate the following
   (a) $27 \div 4$

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   (b) $-32 \div 7$

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   (c) $41 \mod 11$

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   (d) $-43 \mod 13$

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   (e) $\sum_{k=2}^{5} (2k^2 + 3)$

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   (f) $\prod_{m=1}^{5} \frac{2m - 1}{2m + 1}$

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   (g) $[5.48]$

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   (h) $\lfloor -17/3 \rfloor$

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2. (5 Points) Today is a Friday. What day of the week will it be 120 days from now?

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3. (5 Points) Show that if $a | b$ and $a | c$, then $a | (3b - 2c)$.

4. (5 Points) Show that the square of any integer is either of the form $4k$ or $4k + 1$.

5. (5 Points) If $n = 6k + 1$ show that 12 divides $n^2 - 1$. 
6. (5 Points) If \( n \mod 3 = 2 \) then show \( \left\lfloor \frac{n}{3} \right\rfloor = \frac{n + 1}{3} \).

7. (5 Points) What is a formula for the general term \( a_k \) of the sequence that starts
\[
\frac{3}{5}, \frac{-5}{7}, \frac{7}{9}, \frac{-9}{11}, \frac{11}{13}, \ldots
\]

8. (5 Points) Show that the square root of an irrational number is irrational.
9. (5 Points) Either prove or give a counterexample to the statement “the square of an irrational number is irrational”.

10. (10 Points) Write the following using summation or product notation.
(a) \(1^3 - 2^3 + 3^3 - 4^3 + 5^3 - 6^3\)

(b) \(1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 15\)

11. (10 Points) Let \(A = \{b, c, d, f, g\}, B = \{a, b, c\}\) and \(C = \{a, f\}\) the find the following
(a) \(A - B - C\)

(b) \(B - (A \cup B)\)
12. (10 Points) Draw the Venn diagrams for the following
   (a) $A \cup (B \cap C)$

   (b) $A^c - (B \cup C)$

13. (5 points) Prove by induction that for $n \geq 0\ 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$. 