Mathematics 174 Final Name: ________________________

Show your work to get credit. An answer with no work will not get credit.

1. (40 Points) Evaluate the following

   (a) $-42 \, div \, 9$ ________________

   (b) $75 \, mod \, 12$ ________________

   (c) $\displaystyle\sum_{j=3}^{7}(4-3j)$ ________________

   (d) $\displaystyle\prod_{\ell=2}^{5}\frac{\ell+2}{2\ell+1}$ ________________

   (e) $\left\lceil \dfrac{-11}{3} \right\rceil$ ________________

   (f) $P(8,3)$ ________________

   (g) $\dbinom{7}{5}$ ________________

   (h) Write 89_{10} in base 2 ________________

   (i) Write 1100111_2 in base 10 ________________

   (j) Change $12.1212121212\ldots$ to a ratio of integers. ________________

2. (5 Points) Make a truth table for $(p \wedge q) \vee (\sim p \wedge \sim q)$.

p	q	

3. (10 Points) Write out the negations of the following sentences:
   (a) If I read the text, then I will get an A.

   (b) Every toad is slimy.

4. (5 Points) Is the following argument valid? Justify your answer.

   This apple is red or it green.
   This apple is not green.
   $\therefore$ This apple is red.

answer ________________________

Justification:

5. (10 Points) Assume that m and n are integers. Then justify your answers to the following questions:
   (a) Is $4m^3n + 6m^4 + 7$ odd? answer ______________________
   Justification:

   (b) Is $30mn - 15n^4 - 9$ divisable by 3? answer ______________________
   Justification:

6. (5 Points) Show that for any integer n that $n(n+3)$ is even.

7. (10 Points) Prove that if a, b, and c are any integers and $a \mid b$ and $a \mid c$ then $a \mid (b^2 + c)$.

8. (5 Points) If $n \, mod \, 5 = 3$ then show $\left\lfloor \frac{n}{5} \right\rfloor = \frac{n-3}{5}$.

9. (5 Points) What is a formula for the general term a_k of the sequence that starts

$$\frac{3}{1}, \frac{-4}{1 \cdot 2}, \frac{5}{1 \cdot 2 \cdot 3}, \frac{-6}{1 \cdot 2 \cdot 3 \cdot 4}, \frac{7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}, \frac{-8}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}, \cdots$$

$a_n =$ ____________________

10. (5 Points) Write the following using summation notation: $2^7 + 3^7 + 4^7 + \cdots + n^7$.

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11. (10 Points) Let $A = \{2, 3, 4, 6\}$, $B = \{1, 2, 3\}$ and $C = \{1, 4\}$ the find the following

(a) $A - B - C$ ________________

(b) $(A \cap B) \cup C$ ________________

12. (5 Points) Draw the Venn diagrams for $(A \cup B) - C$.

13. (10 points) Passwords for a computer are to be length 7 and are composed of the 26 lower case letters of the alphabet and the 10 digits. (That is things like $c7ry49z$.)

(a) How many such passwords are there? ________________

(b) How many such password are there if it is required that they are 4 letters followed by 3 digits?

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14. (5 Points) Two standard dice are rolled. What is the probably of the sum of the dice being 9?

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15. (10 points) A bakery produces 8 types of donuts.

    (a) How many different selections of 12 donuts are there?

    ____

    (b) Hom many different selections of 12 donuts are there if at least four are to be plain?

    ____

16. (10 points) Solve the following second order recursion relation $a_k = 6a_{k-1} - 9a_{k-2}$ with initial conditions $a_0 = 3$, $a_1 = 5$.

    $a_n =$ ______________________________

17. (5 points) This is the five free points I promised you.

Extra Credit Problems: (These will be graded with no partial credit, so make sure you have done the other problems before trying these.)

1. (5 Points) Use induction to show that

$$1 + 3 + 3^2 + \cdots + 3^n = \frac{1}{2}(3^{n+1} - 1).$$

2. (5 Points) Prove that $\sqrt{3}$ is irrational.

You have been a fun group to work with. Have a nice holiday.