MATH 142 **More on Improper Integrals** **Spring, 1996**

1. Compute, if possible, the following integrals.

   a. $\displaystyle\int_1^\infty \frac{1}{x^{1.01}}\,dx\,.$

   b. $\displaystyle\int_1^\infty \frac{1}{x^{0.99}}\,dx\,.$

   c. $\displaystyle\int_0^\infty \cos(x)\,dx\,.$

   d. $\displaystyle\int_{-\infty}^0 e^{3x}\,dx\,.$

   e. $\displaystyle\int_2^\infty \frac{x}{(1+x^2)^2}\,dx\,.$

   f. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx\,.$

   g. $\displaystyle\int_{-\infty}^{-1} xe^{-x^2}\,dx\,.$

   h. $\displaystyle\int_{\sqrt{3}}^\infty \frac{1}{1+x^2}\,dx\,.$

2. Compute, if possible, $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$, where:

   a. $p > 1\,.$

   b. $p = 1\,.$

   c. $p < 1\,.$

3. Find the area of the region under the curve $y = \dfrac{2}{4x^2-1}$ to the right of $x = 1\,.$

4. Compute, if possible, the following integrals.

a. $$\int_3^4 \frac{1}{\sqrt{4-x}}\,dx\,.$$

b. $$\int_0^1 \ln x\,dx\,.$$

c. $$\int_0^2 \frac{1}{x^4}\,dx\,.$$

d. $$\int_{-3}^2 \frac{1}{x^4}\,dx\,.$$

e. $$\int_1^2 \frac{1}{(x-1)^{1/3}}\,dx\,.$$

f. $$\int_2^4 \frac{1}{(3-x)^{2/3}}\,dx\,.$$

g. $$\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx\,.$$

h. $$\int_{-3}^0 \frac{x}{(x^2-4)^{2/3}}\,dx\,.$$

i. $$\int_0^{\pi/4} \tan(2x)\,dx\,.$$

j. $$\int_0^1 \frac{\ln x}{x}\,dx\,.$$

5. **The Paradox of Gabriel's Horn:** Let the curve $y = \frac{1}{x}$ on $[1, \infty)$ be revolved about the x-axis, thereby generating a surface called Gabriel's Horn.

a. Graph this surface.

b. Show that the volume of this horn is finite.

c. Show that the surface area of this horn is infinite.

d. Parts a. and b. seem to say that the horn can be filled with a finite amount of paint, and yet there is not enough to paint its inside surface! Is this possible?