Test 3 Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (25 points) Compute the following:

(a) $\int \frac{3t}{t^2+t-2}\,dt$

(b) $\int_0^\infty xe^{-2x}\,dx$

(c) $1000 + 1000(1.05) + 1000(1.05)^2 + \cdots + 1000(1.05)^{29}$

(d) $x - x^4 + x^7 - x^{10} + x^{13} - x^{16} + \cdot$

(e) $\lim_{x\to 0} \frac{e^{2x} - 1 - 2x}{x^2}$

2. (10 points) Let T be rotation by $\pi/2$ around the origin.
(a) What is the matrix for T?

(b) What is the image of the point $3\mathbf{i} + 2\mathbf{j}$ under T?.

3. (10 points) Solve the initial value problem $\frac{dV}{dt} = CV^{\frac{2}{3}}$, $V(0) = 8$ (here C is a constant).

4. (15 points) (a) Find the first four nonzero terms of the series for $f(x) = \dfrac{\sin(2x)}{x}$ at the point $x = 0$.

(b) Find the first three nonzero terms of $H(t) = \dfrac{1}{2t+1}$ at the point $t = 2$.

5. (10 points) A ball is dropped straight down form a height of 10 feet and keeps bouncing so that each bounce is $\frac{3}{4}$ the height of the bounce before (so that the first bounce is 7.5 feet). What is the total distance the ball covers by the time it stops bouncing?

6. (15 points) The probability that a light bulb burns out during its first t weeks of use has the probability density function

$$p(t) = \begin{cases} \frac{1}{50} e^{\frac{-x}{50}} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

(a) What is the probability that a light bulb lasts 150 weeks?

(b) What it the expected value for the life time of a bulb?

7. (15 points) On a cold day a stone is brought inside. By Newton's Law of cooling the stone warms so that the rate of change of its temperature is proportional to its difference in temperature with the air in the room. If the air inside is at a temperature of 65°F and the stone has a temperature of 30°F when it is first brought in then

(a) Write a rate equation satisfied by the temperature of the stone.

(b) If after 20 minutes the stone is 40°F then what is its temperature after an hour?