

Test 1

Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (15 points) Find the derivatives to the following. You do not have to simplify your answers, but if you do you are responsible for any errors you make.

(a) $f(x) = \arctan\left(\frac{x}{3}\right)$
 $f'(x) =$

(b) $\theta(t) = \arcsin(t + 2)$
 $\theta'(t) =$

(c) $y = e^{3x^4+x} - 9$
 $y' =$

(d) $f(x, y) = \sqrt{x - y^2}$
 $\frac{\partial f}{\partial x} =$
 $\frac{\partial f}{\partial y} =$

2. (15 points) Find the antiderivatives of the following:

(a) $h(t) = 3t^4 - 9t^2 + 1$
 $H(t) =$

(b) $f(x) = 6e^{3x} + 9$
 $F(x) =$

(c) $g(s) = 2(9)^s$
 $G(s) =$

(d) $v(r) = 3r \cos(r^2)$
 $V(r) =$

3. (10 points) Find the maximum and minimum values of $y = x^3 - 6x^2 + 3$ on the interval $[-2, 5]$. At what point(s) do the maximum and minimum occur?

4. (10 points) Sketch the graph of the solution to $y' = \frac{x(x-2)}{1+y^2}$, $y(1) = 2$.

5. (10 points) Give upper and lower bounds on the area under the following graph.

6. (15 points) The Forestry service employs student volunteers to plant trees. These students each plant 20 trees per hour. On one of the days the number of students working as a function of time t in hours (starting at 6:00AM) is

$$S(t) = \begin{cases} 3, & 0 \leq t < 4 \\ 1, & 4 \leq t < 5 \\ 4, & 5 \leq t \leq 8. \end{cases}$$

(a) Graph how the number of staff hours accumulate as a function of time:

(b) What was the total number of trees planted by the students?

7. (10 points) (a) Compute $\sum_{k=2}^6 (2k - 1)$

(b) Write $3^2 + 1 + 4^2 + 1 + 5^2 + 1 + 6^2 + 1 + 7^2 + 1$ in \sum notation.

8. (10 points) Derive a formula for the derivative of $y = \operatorname{arccot}(x)$ (this is the function so that $\cot(y) = x$. (HINT: You may want to use the identity $\csc^2(y) = 1 + \cot^2(y)$.)

9. (15 points) A farmer has a 120 yards of fencing and plans to make a pen with a divider parallel to one of the sides as shown. What are the dimensions of the pen that will maximize the total area on the pen?