Review for Test 1

1. **Things related to last term.** First of all you are responsible for everything from last term. In particular you should be able to compute derivatives of common functions. Along these lines we have leaned the derivatives of some new functions this term. They are

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}, \quad \frac{d}{dx}\arccos(x) = \frac{-1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

Recall that we had a homework where you derived these formulas, so it be reasonable to be expected to do this on the test. For example to show the derivative formula for $\arctan(x)$ proceed as follows.

> Let $y = \arctan(x)$, then we wish to compute y'. Form the definition of arctan it follows $\tan(y) = x$. Take d/dx of this and use the chain rule and the identity $\sec^2 = 1 + \tan^2$ to get
>
> $$1 = \frac{d}{dx}x = \frac{d}{dx}\tan(y) = \sec^2(y)y' = (1+\tan^2(y))y'.$$
>
> But $\tan(y) = x$ so we can solve for y' and get
>
> $$\frac{d}{dx}\arctan(x) = y' = \frac{1}{1+\tan^2(y)} = \frac{1}{1+x^2}.$$

Anther fact from last term you should know is that the solution to the initial value problem $y' = ky$, $y(0) = C$ is $y = Ce^{kx}$.

2. **Euler's method and differential equations.** Euler's method for approximating the solution to a differential equation is to use the several steps in the microscope approximation $\Delta y \approx f'(a)\Delta x$. As a practice problem do the following: Approximate $y(2)$ if $y' = 1 + xy$, $y(1) = -1$ by taking four steps starting at $x = 1$.

3. **Graphing Solutions to Differential Equations.** You should be able to give a sketch of the graph of a differential equation without having to solve it. As same problems graph the solutions to the following (a) $y' = x(2-x)/(1+y^2)$, $y(1) = 3$ showing all local maxima and minima. (b) Graph the solution to $y' = y(5-y)$, $y(0) = 3$.

4. **Graphing and the first and second derivatives of a function.** Look over the problems on using the first and second derivatives to graph a function. Problems like the homework and the ones we did in class are fair game.

5. **Maximum and minimum problems.** You should expect a word problem where you have to find the maximum or minimum of a function. Again the homework problems give a good sample of what to expect.

6. **Chapter Six.** While I have this listed last, it will be a big part of the test. The test will cover sections §6.1, §6.2 and §6.3. As this part of the book is very well written there is no need to go into much detail here. Sample problems would be: Pages 340–341 1–3. Page 364–365 16, 17, 18 (these have to do with Σ notation. If you have questions about them be sure to ask before Tuesday). You will also have to do a problem like our jogger problems, where you are given the speed and have to approximate the distance covered by using Riemann sums. Page 189 1, 2, 3(when doing this problem use the bound

$$\text{error} \leq |f(b) - f(a)|$$

given in class and on page 384 of the test.