Final Name:__________________________

Show your work! Answers that do not have a justification will receive no credit.

1. (60 points) Compute the following:

(a) $f(x) = \sqrt{x^2 + x}$

$f'(x) =$

(b) $h(t) = te^{t^2+t}$

$h'(t) =$

(c) $H(u, v) = \dfrac{\cos(2u)}{2 + \sin(v)}$

$\dfrac{\partial H}{\partial u} =$

$\dfrac{\partial H}{\partial v} =$

(d) $\int (x^3 - 9x^2 + 2x + 1)\, dx$

Answer =

(e) $\int (v^2 + 2v)\cos(av)\, dv$ (where a is a constant).

Answer =

(f) $\int s e^{3s^2+1}\, ds$

Answer =

(g) Find the sum of $50 + 50(1.1) + 50(1.1)^2 + 50(1.1)^3 + \cdots + 50(1.1)^{39}$.
Answer =

(h) Find the sum of $2x - 4x^3 + 8x^5 - 16x^7 + 32x^9 - + \cdots$

Answer =

(i) $\lim_{x \to \pi/2} \dfrac{\cos(x)}{x - \pi/2} =$

(j) The first three nonzero terms of the series centered at $x = 0$ for the function $\dfrac{\cos(2x) - 1}{x}$

Answer =

(k) $\displaystyle\int_0^4 \frac{3}{\sqrt{4-p}}\,dp =$

(l) $\displaystyle\int_0^\infty te^{-4t}\,dt =$

2. (15 points) Let $\mathbf{v} = (2, -1)$ and $\mathbf{w} = (1, 3)$. Then find:
(a) $2\mathbf{v} + \mathbf{w}$ and make a graph showing $\mathbf{v}$, $\mathbf{w}$ and $2\mathbf{v} + \mathbf{w}$

(b) $\mathbf{v} \cdot \mathbf{w} =$

(c) The unit vector in the direction of $\mathbf{w} =$

(d) The angle between $\mathbf{v}$ and $\mathbf{w} =$

(e) Scalars r and s so that $r\mathbf{v} + s\mathbf{w} = (5, -4)$

$r =$

$s =$

3. (10 points) Find the maximum and minimum of $h(t) = t^2e^{-t}$ on the interval $[0, 10]$.

Maximum=

Minimum=

4. (10 points) Sketch the graph of the solution to the initial value problem

$$y' = (1 - x^2)e^{-y^2+x^2}, \qquad y(0) = 1$$

showing where all the local maxima and minima occur.

5. (10 points) Solve the initial value problem

$$y' = (2x + 1)y, \qquad y(0) = 4.$$

Answer=

6. (15 points) A survey shows that if a vendor at a small ball park sells hot dogs at x cents each then he will sell $300 - 2x$ of them. At what price should he sell them to maximize his profit?

Answer=

7. (20 points) A particle moves so that its position is $\mathbf{r}(t) = (1 - t)\mathbf{i} + (2t^2 + 1)\mathbf{j}$. Find the following

(a) Velocity=

(b) The speed=

(c) The acceleration=

(d) The length as of the part of the curve between $t = 1$ and $t = 3$ (setting up the integral correctly is worth most of the credit on this one.)

Answer=

(e) The point where the particle cross the y-axis.

Answer=

8. (10 points) For what values of x do the following series converge?
(a) $(x-1) + 2(x-1)^2 + 3(x-1)^3 + 4(x-1)^4 + \ldots + n(x-1)^n + \cdots$.

Answer=

(b) $5 + 5(2x) + 5(2x)^2 + 5(2x)^3 + 5(2x)^4 + \cdots$.

Answer=

9. (10 points) Let T be reflection in the line $y = x$.

(a) What is the matrix for T?

Answer=

(b) What is the image of the point $3\mathbf{i} + 2\mathbf{j}$ under T?.

Answer=

10. (10 points) In polar coordinates plot the following

(a) $(r, \theta) = (2, \pi/4)$, $(r, \theta) = (-3, \pi/2)$,

(b) The curves $r = 4$, and $\theta = 3\pi/2$.

11. (15 points) Our favorite jogger starts out on a run with a thunder storm coming into town. The probability that it starts to rain t minutes after she starts has density function

$$p(t) = \begin{cases} 0 & t < 0 \\ \dfrac{C}{(t+10)^3} & t \geq 0 \end{cases}$$

for some constant C.

(a) What is the value of the constant?

Answer=

(b) What is the cumulative distribution function

Answer=

(c) What is the probability she runs for ten minutes without getting rained on?

Answer=

12. (15 points) 200 guppies are released in Lake Murray. Assume that the rate on increase of the number of guppies is proportional to the number of guppies in the lake.

(a) Write a rate equation and initial condition for the number of guppies in the lake labeling all the variables.

(b) If six months after the they are released there are 500 guppies in the lake, then how long is it before there are 100,000 guppies?

Answer=