Mathematics 141 Test #2 Name:________________________

Show your work to get credit. An answer with no work will not get credit.

(1) (25 points) Compute the following (knowing what the notation means is part of the problem).

(a) $\dfrac{dy}{dx}$ where $y = \sin(x^2)$ $\qquad \dfrac{dy}{dx} =$ ________________

(b) $\dfrac{ds}{dt}$ where $s = 5(2t^3 + t)^{50}$. $\qquad \dfrac{ds}{dt} =$ ________________

(c) $f''(x)$ where $f(x) = 5x^3 - 4x^2 + 7x - 9$ $\qquad f''(x) =$ ________________

(d) $\dfrac{d}{dt}\left(t^2 \cos^2(t) - 3t\right)$ $\qquad$ ________________

(e) $\dfrac{d^2y}{dx^2}$ where $y = \dfrac{2}{x^5}$ $\qquad \dfrac{d^2y}{dx^2} =$ ________________

(f) $\dfrac{ds}{dt}$ where $s = \dfrac{4}{(1 + \cos(t))^2}$ $\qquad \dfrac{ds}{dt} =$ ________________

(g) $\dfrac{dA}{dr}$ where $A = 4\sqrt{2r^3 + r^2}$ $\qquad \dfrac{dA}{dr} =$ ________________

(2) (5 points) If x and y are related by $x^2 + 4xy + y^2 = 9$ find $\dfrac{dy}{dx}$ by implicit differentiation.

$\dfrac{dy}{dx} =$ ________________

(3) (5 points) Find the tangent line to $2xy^2 + xy = 5$ at the point $(1, 2)$.

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(4) (10 points) A 20 foot long ladder is leaning against the side of a building, but the base is slipping away from the building at 5 ft/sec. How fast is the top of the latter moving when it is 16 feet from the ground?

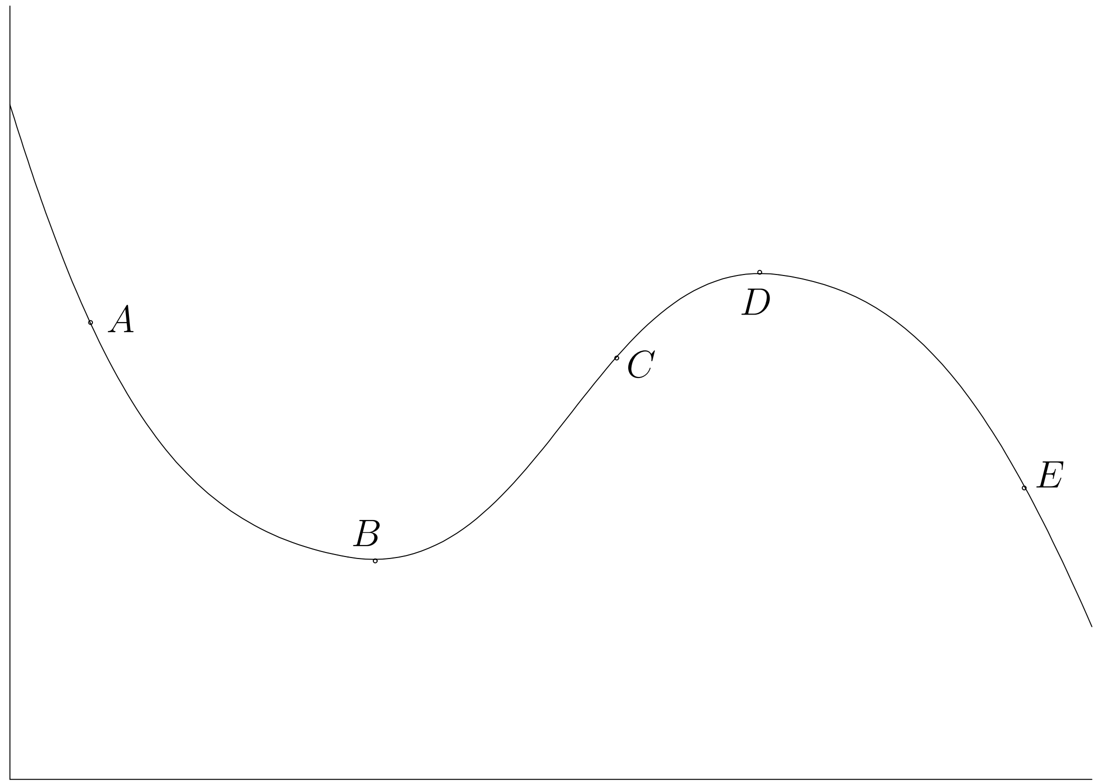
Rate top is moving = ________________________

(5) (10 points) Draw graphs of functions $f(x)$ with the following properties.

(a) $f'(x) > 0$ and $f''(x) < 0$.

(b) $f(1) = 2$, $f'(1) = 0$, and $f''(x) > 0$.

(6) (5 points) In the following figure



(a) At which of the labeled points is $f' > 0$? ________________

(b) Ate which of the labeled points if $f' = 0$ ________________

(c) At which of the labeled points is $f'' > 0$ ________________

(d) At which of the labeled points is $f'' < 0$ ________________

(7) (10 points) Find the maximum and minimum of $y = 6x^2 - x^3$ on the interval $[-1, 7]$.

Maximum = ________________

Maximizer = ________________

Minimum = ________________

Minimizer = ________________

(8) (10 points) Sketch the graph, labeling all the local maximums, local minimums and inflection points of a function $y = f(x)$ on $[1, 4]$ with the following properties:

- $f' > 0$ on the intervals $(1, 2)$ and $(3, 4)$,
- $f' < 0$ on the interval $(2, 3)$,
- $f'' < 0$ on $(1, 2.5)$,
- $f'' > 0$ on $(2.5, 4)$, and
- $f(1) = 3$, $f(2) = 6$, $f(3) = 5$, $f(4) = 9$.

(9) (15 points)

(a) Where is the function $f(x) = x^3 - 12x$ increasing and where is it decreasing?

Increasing ____________________

Decreasing ____________________

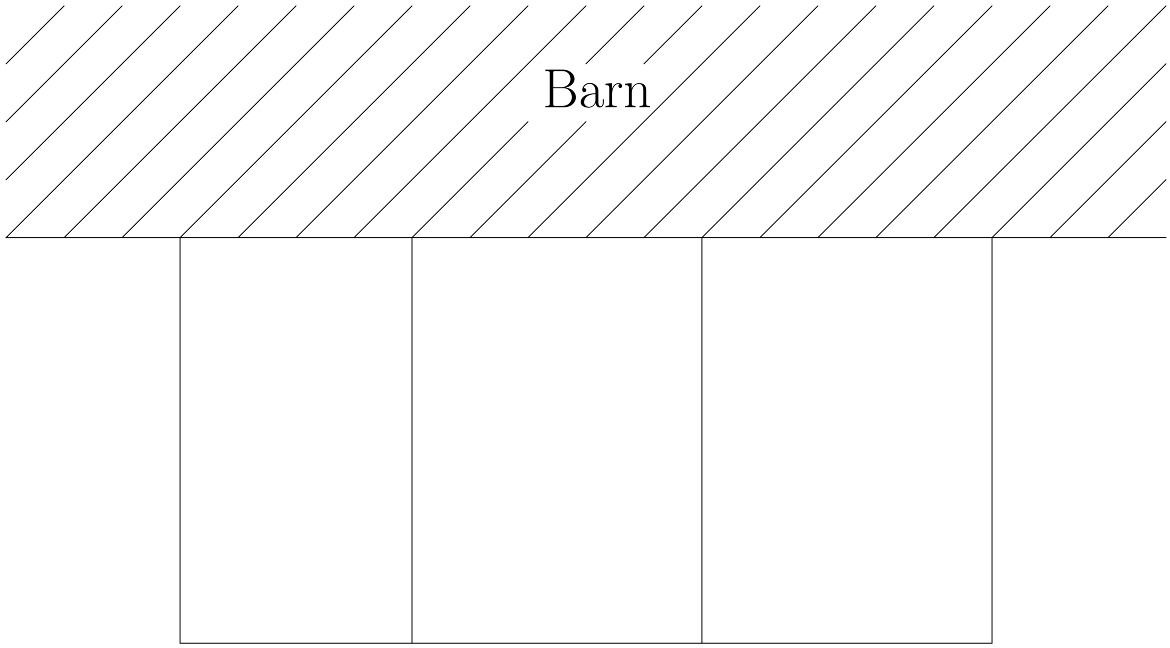
(b) Where is the function $g(t) = t^3 - 6t^2 + 7t - 9$ concave up, and where is it concave down. Are there any inflection points.

Concave up ____________________

Concave down ____________________

Infection points ____________________

(10) (10 points) A farmer has 80 feet of fencing and he wishes to build a pen against the side of a barn divided into three smaller pens as shown. If no fencing is needed on side against the barn, what are the dimensions of the pen that incloses the largest area.



Length of side parallel to barn ____________________

Length of sides orthogonal to barn ____________________