Mathematics 141 Test #1 Name:______________________________

Show your work to get credit. An answer with no work will not get credit.

(1) (10 points) Compute the following limits:

(a) $\lim_{x \to 2} \frac{3x+1}{x^2-6} =$

(b) $\lim_{t \to 0} \frac{2\cos(2t)}{4+\sin(2t)} =$

(c) $\lim_{h \to 0} \frac{(3+h)^2-3^2}{h} =$

(d) $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} =$

(e) $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} =$

(f) $\lim_{x \to \infty} \frac{2x^2+7x-9}{3x^2-5x+2} =$

(g) $\lim_{x \to -\infty} \frac{4x+1}{x^4+16} =$

(h) $\lim_{x \to 0} 4x\cot(3x) =$

(i) $\lim_{t \to 3^-} \frac{t^2+7}{t-3} =$

(2) (40 points) Compute the following derivatives. You do not have to simplify your answers.

(a) $y = 5x^4 - 7x^3 + 4x^2 + 5x - 9$

$y' =$

(b) $y = 7x^{-4} + 5\pi^{-3}$

$y' =$

(c) $C(q) = \dfrac{5}{q^3} - \dfrac{4}{q^4}$

$C'(q) =$

(d) $y' = 5\sqrt{x} - \sqrt[3]{x}$

$y' =$

(e) $y = (x^2 + 1)(4x^3 + 2)$

$y' =$

(f) $y = 3(x^3 + 2)^2$

$y' =$

(g) $y = \dfrac{3}{x^2 + x + 1}$

$y' =$

(h) $w = (z^2 + 1)(\sqrt{z} + 3)$

$w' =$

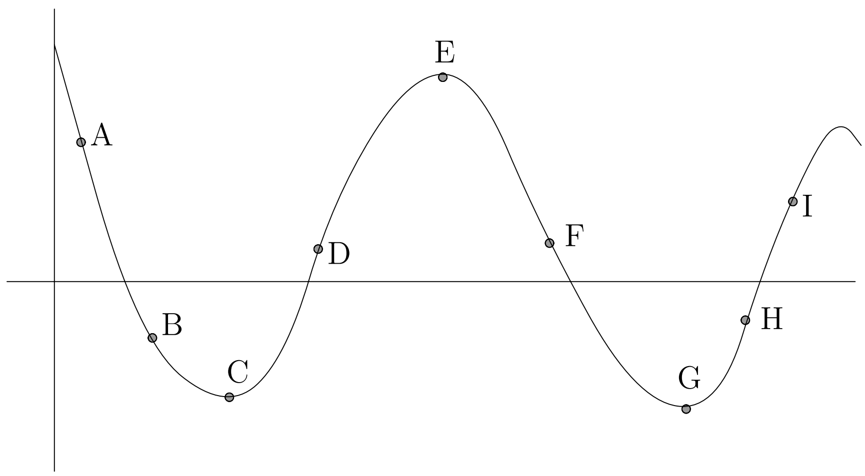
(i) $R(t) = \dfrac{2t^3 + t}{t^2 + 3t}$

$R'(t) =$

(j) $y = (x + 1)(x^2 + 1)(x^3 + 1)$

$y' =$

(3) (5 points) Let $y = f(x)$ have the following graph.

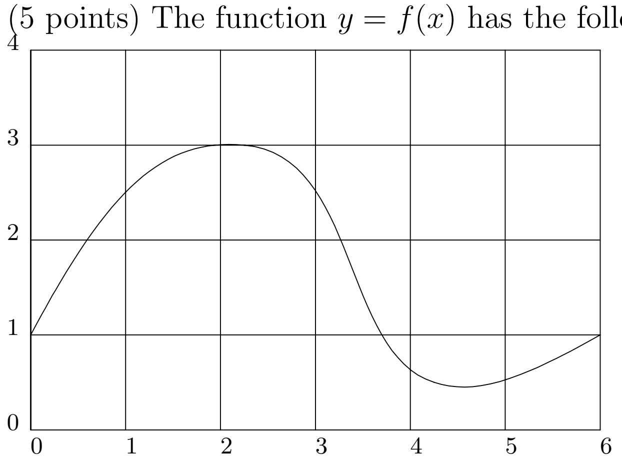


(a) At which of the labeled points is $f'(x) > 0$? ____________

(b) At which is the labeled points is $f'(x) < 0$? ____________

(c) At which is the labeled points is $f'(x) = 0$? ____________

(4) (5 points) The function $y = f(x)$ has the following graph. Estimate the derivative $f'(3)$.



$f'(3) \approx$ ____________

(5) (5 points) What is the equation of the tangent line to $y = x^2 + x - 1$ at the point where $x = 2$?

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(6) (15 points)

(a) State what it means for a function $f(x)$ to be continuous at the point $x = a$.

(b) State the Itermediate Value Theorem.

(c) Show that the equation $2x^3 + x - 5 = 0$ has at least one solution between in the interval $[1, 2]$.

(7) (20 points)

(a) Let f be a function and a a real number $h \neq 0$. Explain the geometric meaning of the difference quotient $\dfrac{f(a+h) - f(a)}{h}$ (include a picutre).

(b) State the definition of the derivatice $f'(a)$ as a limit.

(c) Use your answers to (a) and (b) to explain why $f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x = a$.

(d) Use the limit definition of derivative to find a formula for $f'(a)$ when $f(x) = \sqrt{2x+1}$.

$f'(a) =$ ____________________

(8) (5 points) For the function $y = f(x)$ with graph below answer the following.

(a) What is $\lim_{x \to 2} f(x)$ ____________________

(b) What is $\lim_{x \to 2^-} f(x)$ ____________________

(c) What is $\lim_{x \to 2^+} f(x)$ ____________________