Mathematics 141 Test #1 Name:______________________________

Show your work to get credit. An answer with no work will not get credit.

(1) (15 points) Compute the following limits:

(a) $\displaystyle\lim_{x\to 3}\frac{2x-1}{x^2+4} =$

(b) $\displaystyle\lim_{t\to 0}\frac{\cos(2t)}{3+\sin(t)} =$

(c) $\displaystyle\lim_{h\to 0}\frac{(2+h)^2-2^2}{h} =$

(d) $\displaystyle\lim_{\theta\to 0}\frac{\sin\theta}{\theta} =$

(e) $\displaystyle\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} =$

(f) $\displaystyle\lim_{x\to\infty}\frac{3x^2+x-9}{4x^2-3x+7} =$

(g) $\displaystyle\lim_{x\to 0} 4x\cot(3x) =$

(2) (45 points) Compute the following derivatives. You do not have to simplify your answers.

(a) $y = 6x^5 - 2x^3 + 7x^2 - 6x + 3$

$y' =$

(b) $y = 5x^{-3} + 4\pi^{-2}$

$y' =$

(c) $C(q) = \dfrac{5}{q^3} - \dfrac{4}{q^4}$

$C'(q) =$

(d) $y = \cos(x)$

$y' =$

(e) $y = \sin(x)$

$y' =$

(f) $y = \tan(x)$

$y' =$

(g) $y = \sec(x)$

$y' =$

(h) $w = 3\sqrt{z}$

$w' =$

(i) $P(t) = 3t^2 \sin(t)$

$P'(t) =$

(j) $R(t) = \dfrac{2t^3 + t}{t^2 + 3t}$

$R'(t) =$

(k) $y = 7(x^4 - 3x^2 + 6)^{11}$

$y' =$

(l) $y = 3\cos(x^4)$

$y' =$

(m) $y = 4(x + \tan(2x))^3$

$y' =$

(n) $Q(t) = \dfrac{1 + \cos(2t)}{1 + \sin(2t)}$

$Q'(t) =$

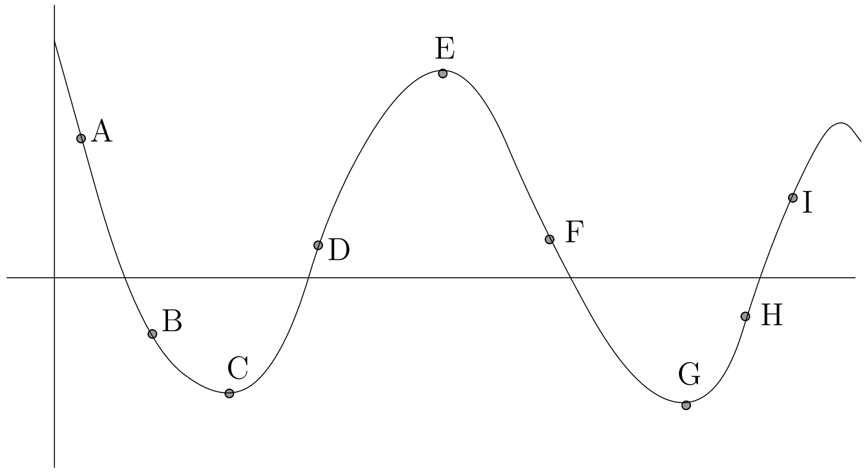
(o) $y = 4\left(\dfrac{x+2}{x+1}\right)^5$

$y' =$

(p) $y = \sqrt{x^2 + \cos^2(3x)}$

$y' =$

(3) (10 points) Let $y = f(x)$ have the following graph.

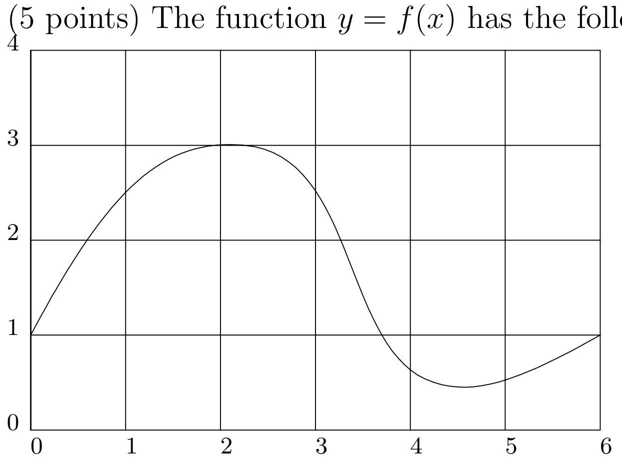


(a) At which of the labeled points is $f'(x) > 0$? ________________

(b) At which is the labeled points is $f'(x) < 0$? ________________

(c) At which is the labeled points is $f'(x) = 0$? ________________

(4) (5 points) The function $y = f(x)$ has the following graph. Estimate the derivative $f'(3)$.



$f'(3) \approx$ ________________

(5) (5 points) What is the equation of the tangent line to $y = x^2 + 3x - 2$ at the point where $x = -1$?

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(6) (10 points)

(a) State what it means for a function $f(x)$ to be continuous on an interval I.

(b) State the Itermediate Value Theorem.

(c) Show that the equation $2x^3+x-5=0$ has at least one solution between in the interval $[1, 2]$.

(7) (10 points)

(a) Let f be a function and a a real number $h \neq 0$. Explain the geometric meaning of the difference quotient $\dfrac{f(a+h)-f(a)}{h}$ (include a picutre).

(b) State the definition of the derivatice $f'(a)$ as a limit.

(c) Use your answers to the last two questions to explain why $f'(a)$ is the slope of the tangent line to $y = f(x)$ at $x = a$.