Mathematics 141 Final Name:____________________________

Show your work to get credit. An answer with no work will not get credit.

(1) (28 points) Compute the following derivatives. You do not have to simplify your answers.

(a) $y = 3x^5 - 2x^4 + 7x^3 - 4x^2 + 3x - 7$

$y' =$

(b) $y = \dfrac{4}{x^3} + \dfrac{5}{\pi^2}$

$y' =$

(c) $A(t) = \dfrac{4}{t^3} - \dfrac{7}{t^6}$

$A'(t) =$

(d) $y = \cos(x)$

$y' =$

(e) $y = \tan(2x)$

$y' =$

(f) $y = \csc(3x)$

$y' =$

(g) $y = 3x^{3/2} + \sqrt{x}$

$y' =$

(h) $P(t) = 4t^7 \cos(t)$

$P'(t) =$

(i) $R(t) = \dfrac{1 + \sin(t)}{1 - \sin(t)}$

$R'(t) =$

(j) $y = \cos^3(2x + 1)$

$y' =$

(k) $R(t) = 7(t^3 + t + 1)^{11}$

$R'(t) =$

(l) $A(\theta) = \dfrac{2}{\sqrt{3+\cos(2\theta)}}$

$A'(\theta) =$

(m) $G(x) = \displaystyle\int_1^x \cos(t^2)\,dt$

$G'(x) =$

(n) $F(x) = \displaystyle\int_1^{x^2} \sin(t^3)\,dt$

$F'(x) =$

(2) (21 points) Compute the following antiderivatives.

(a) $\displaystyle\int (4x^3 - 9x^2 + 7x - 3)\,dx$ ____________________

(b) $\displaystyle\int \left(\frac{2}{\sqrt{s^3}} - \frac{2}{\sqrt{\pi^3}}\right) ds$ ____________________

(c) $\displaystyle\int \frac{w^2 - 2w + 3}{\sqrt{w}}\,dw$ ____________________

(d) $\displaystyle\int (-2\cos t + 3\sin t)\,dt$ ____________________

(e) $\displaystyle\int (\cos(3\theta) - \sin(4\theta))\,d\theta$ ____________________

(f) $\displaystyle\int \frac{x^3}{\sqrt{x^4+1}}\,dx$ ____________________

(g) $\displaystyle\int 2\sin^3(4t)\cos(4t)\,dt$ ____________________

(3) (16 points) Compute the following definite integrals.

(a) $\displaystyle\int_{-1}^{2} (6x^2 - 4x + 3)\,dx$ ____________________

(b) $\displaystyle\int_0^{\pi/2} \sin 5t\,dt$ ____________________

(c) $\displaystyle\int_0^1 \frac{x}{(x^2+1)^2}\,dx$ ____________________

(d) $\displaystyle\int_0^1 \frac{x+2}{\sqrt{x^2+4x+5}}\,dx$ ____________________

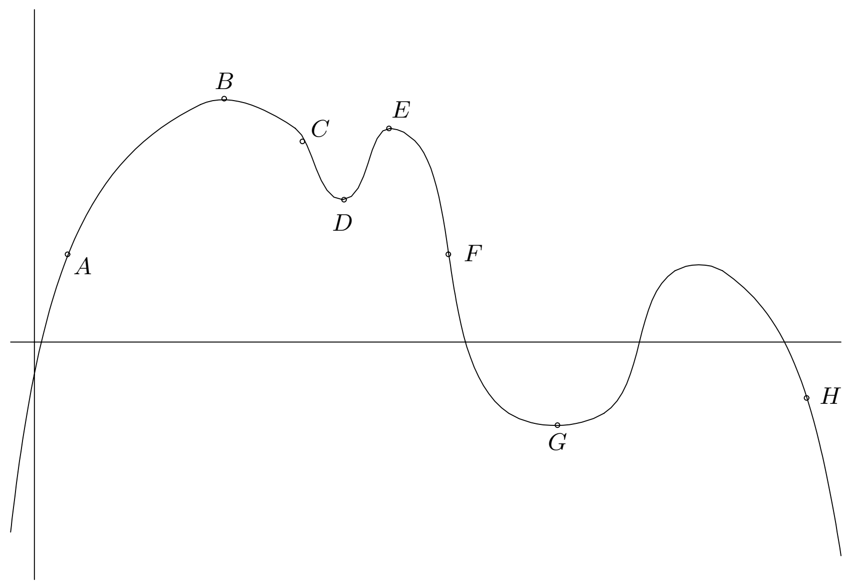
(4) (5 points) Compute the following limits.

(a) $\displaystyle\lim_{x\to 3} \frac{x^2-5x+6}{x-3} =$

(b) $\displaystyle\lim_{x\to 0} \frac{\sin(4x)}{5x} =$

(c) $\lim_{x \to \infty} \dfrac{3x^2 - 4x + 7}{5x^2 - 3x + 4} =$

(5) (5 points) Let $y = f(x)$ have the following graph.



(a) At which points is $f'(x) > 0$? ________________

(b) At which points is $f'(x) < 0$? ________________

(c) At which points is $f'(x) = 0$? ________________

(d) At which points does f have a local maximum? ________________

(e) At which points does f have a local minimum? ________________

(6) (5 points)

(a) State the definition of derivative in terms of a limit.

(b) State the mean value theorem.

(7) (5 points) Compute the first three derivatives of $f(x) = x\cos(x)$.

$f'(x) =$

$f''(x) =$

$f'''(x) =$

(8) (5 points) If x and y are related by

$$x^2 + xy + 3y^2 + x - y = 4$$

then find $\frac{dy}{dx}$ by implicit differentiation.

$\frac{dy}{dx} =$ ____________________

(9) (5 points) Find the tangent line to $x^2 + 2xy + 4y^2 = 21$ at the point $(1, 2)$.

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(10) (5 points) A circular oil spill spreads over a lake so that the area increasing at $10\text{ft}^2/\text{hour}$. At what rate is the radius increasing when the the radius is 25 feet?

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(11) (10 points) For the function $f(x) = 2x^3 - 24x + 5$ on $[-4, 3]$, find the critical points, the maximum, the minimum, and the inflection points.

Critical Points: ____________________

Maximum ____________________

Minimum ____________________

Inflection points: ____________________

(12) (5 points) What is the area of largest rectangular pen that can be made with 40 feet of fencing.

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(13) (10 points) For the function $f(x) = -2x^3 + 24x + 2$ find the intervals on with it is increasing and decreasing and the intervals where it is concave up and concave down, and its inflection points.

Increasing ____________________

Decreasing ____________________

Concave up ____________________

Concave down ____________________

Inflection points ____________________

(14) (5 points) If $f'(x) = 4x - 3$ and $f(3) = 2$, then what is $f(x)$?

$f(x) =$ ______________________

(15) (10 points) Graph both $y = x^2 - 3x$ and $y = 3 - x$ on the same graph showing the points of intersection of the curves. Then find the area bounded between them.

Area = ______________________

Graph:

(16) (5 points) What is the volume when the region bounded by the lines $y = 0$, $y = x$ and $x = 1$ is revolved about the y-axis?

Volume = ______________________

(17) (5 points) What is the length of the graph of $y = \frac{2}{3}x^{3/2}$ from $x = 3$ to $x = 15$?

Length = ______________________

Have a good summer.