(1) (28 points) Compute the following derivatives. You do not have to simplify your answers.

(a) \( y = 3x^5 - 2x^4 + 7x^3 - 4x^2 + 3x - 7 \)

\[ y' = \]

(b) \( y = \frac{4}{x^3} + \frac{5}{\pi^2} \)

\[ y' = \]

(c) \( A(t) = \frac{4}{t^3} - \frac{7}{t^6} \)

\[ A'(t) = \]

(d) \( y = \cos(x) \)

\[ y' = \]

(e) \( y = \tan(2x) \)

\[ y' = \]
(f) \( y = \csc(3x) \)
\[
y' =
\]

(g) \( y = 3x^{3/2} + \sqrt{x} \)
\[
y' =
\]

(h) \( P(t) = 4t^7 \cos(t) \)
\[
P'(t) =
\]

(i) \( R(t) = \frac{1 + \sin(t)}{1 - \sin(t)} \)
\[
R'(t) =
\]

(j) \( y = \cos^3(2x + 1) \)
\[
y' =
\]

(k) \( R(t) = 7(t^3 + t + 1)^{11} \)
\[
R'(t) =
\]
(1) \[ A(\theta) = \frac{2}{\sqrt{3 + \cos(2\theta)}} \]
\[ A'(\theta) = \]

(m) \[ G(x) = \int_1^x \cos(t^2) \, dt \]
\[ G'(x) = \]

(n) \[ F(x) = \int_1^{x^2} \sin(t^3) \, dt \]
\[ F'(x) = \]

(2) (21 points) Compute the following antiderivatives.

(a) \[ \int (4x^3 - 9x^2 + 7x - 3) \, dx \]

(b) \[ \int \left( \frac{2}{\sqrt{s^3}} - \frac{2}{\sqrt{\pi^3}} \right) \, ds \]

(c) \[ \int \frac{w^2 - 2w + 3}{\sqrt{w}} \, dw \]
(d) \[ \int (-2 \cos t + 3 \sin t) \, dt \]

(e) \[ \int (\cos(3\theta) - \sin(4\theta)) \, d\theta \]

(f) \[ \int \frac{x^3}{\sqrt{x^4 + 1}} \, dx \]

(g) \[ \int 2 \sin^3(4t) \cos(4t) \, dt \]

(3) (16 points) Compute the following definite integrals.

(a) \[ \int_{-1}^{2} (6x^2 - 4x + 3) \, dx \]
(b) \[ \int_0^{\pi/2} \sin 5t \, dt \]

(c) \[ \int_0^1 \frac{x}{(x^2 + 1)^2} \, dx \]

(d) \[ \int_0^1 \frac{x + 2}{\sqrt{x^2 + 4x + 5}} \, dx \]

(4) (5 points) Compute the following limits.

(a) \[ \lim_{x \to 3} \frac{x^2 - 5x + 6}{x - 3} = \]

(b) \[ \lim_{x \to 0} \frac{\sin(4x)}{5x} = \]
(c) \( \lim_{x \to \infty} \frac{3x^2 - 4x + 7}{5x^2 - 3x + 4} = \)

(5) (5 points) Let \( y = f(x) \) have the following graph.

(a) At which points is \( f'(x) > 0? \)

(b) At which points is \( f'(x) < 0? \)

(c) At which points is \( f'(x) = 0? \)

(d) At which points does \( f \) have a local maximum?

(e) At which points does \( f \) have a local minimum?

(6) (5 points)

(a) State the definition of derivative in terms of a limit.

(b) State the mean value theorem.
(7) (5 points) Compute the first three derivatives of \( f(x) = x \cos(x) \).

\[
\begin{align*}
  f'(x) &= \\
  f''(x) &= \\
  f'''(x) &=
\end{align*}
\]

(8) (5 points) If \( x \) and \( y \) are related by

\[
x^2 + xy + 3y^2 + x - y = 4
\]

then find \( \frac{dy}{dx} \) by implicit differentiation.

\[
\frac{dy}{dx} = 
\]

(9) (5 points) Find the tangent line to \( x^2 + 2xy + 4y^2 = 21 \) at the point \( (1, 2) \).
(10) (5 points) A circular oil spill spreads over a lake so that the area increasing at 10ft$^2$/hour. At what rate is the radius increasing when the radius is 25 feet?

(11) (10 points) For the function $f(x) = 2x^3 - 24x + 5$ on $[-4, 3]$, find the critical points, the maximum, the minimum, and the inflection points.

Critical Points: 

Maximum 

Minimum 

Inflection points: 
(12) (5 points) What is the area of largest rectangular pen that can be made with 40 feet of fencing.

(13) (10 points) For the function $f(x) = -2x^3 + 24x + 2$ find the intervals on with it is increasing and decreasing and the intervals where it is concave up and concave down, and its inflection points.

<table>
<thead>
<tr>
<th>Increasing</th>
</tr>
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<tbody>
<tr>
<td>Decreasing</td>
</tr>
<tr>
<td>Concave up</td>
</tr>
<tr>
<td>Concave down</td>
</tr>
<tr>
<td>Inflection points</td>
</tr>
</tbody>
</table>
(14) (5 points) If $f'(x) = 4x - 3$ and $f(3) = 2$, then what is $f(x)$?

$$f(x) = \underline{\text{____________________}}$$

(15) (10 points) Graph both $y = x^2 - 3x$ and $y = 3 - x$ on the same graph showing the points of intersection of the curves. Then find the area bounded between them.

Graph:

Area = \underline{\text{____________________}}
(16) (5 points) What is the volume when the region bounded by the lines $y = 0$, $y = x$ and $x = 1$ is revolved about the $y$-axis?

Volume = ______________________

(17) (5 points) What is the length of the graph of $y = \frac{2}{3}x^{3/2}$ from $x = 3$ to $x = 15$?

Length = ______________________

Have a good summer.