1. (30 Points) Take the derivatives of the following functions. In the first blank put the unsimplified form. If there is a second blank then put in the simplified form of the derivative.

(a) \( f(x) = 4x^7 - 9x^4 + 3x^2 - 9 \)
\[ f'(x) = \]

(b) \( w = \sin(\theta) - 3\cos(\theta) + 4\tan(\theta) \)
\[ \frac{dw}{d\theta} = \]

(c) \( P(r) = 9 \cdot 7^r \)
\[ P'(r) = \]

(d) \( M(t) = 4\cos(t^2 + t) \)
\[ M'(t) = \]
(e) \( w = (x^3 - x)\sqrt{x^2 + 9} \)

\[
\frac{dw}{dx} = \frac{(x^3 - x)\frac{2x}{2\sqrt{x^2 + 9}} + (3x^2 - 1)(x^2 + 9)^{1/2}}{x^2 + 9}
\]

\[
= \frac{x^3 - x + 3x^2 - 1}{x^2 + 9}
\]

Simplified form

(f) \( H(t) = \frac{3t^2 + t}{4 - t^2} \)

\[
H'(t) = \frac{(6t + 1)(4 - t^2) - (3t^2 + t)(-2t)}{(4 - t^2)^2}
\]

\[
= \frac{6t^2 + 6t + 4}{(4 - t^2)^2}
\]

Simplified form

(g) \( V(\alpha) = \alpha^4 5^\alpha \sin(2\alpha) \)

\[
V'(\alpha) = \frac{d}{d\alpha} \left( \alpha^4 5^\alpha \sin(2\alpha) \right)
\]

\[
= \alpha^3 5^\alpha \sin(2\alpha) + 4\alpha^4 5^\alpha \cos(2\alpha)
\]

Simplified form
2. (10 Points)
(a) If \( f(2) = 4 \) and \( f'(2) = -3 \) then a reasonable estimate for \( f(2.15) \) is \( 
\). 

(b) If \( G(-2) = 4 \) and \( G'(-2) = -1/2 \) then a reasonable estimate of \( G(-2.2) \) is \( 
\). 

(c) \( F(2) = 0 \) and \( F'(2) = .4 \) then a reasonable estimate of \( F(\quad) \) is \( .15 \).

3. (15 Points) The volume of a can of radius \( r \) inches is \( V(r) = 4r^3 - r^2 \).

(a) Write the microscope equation at the point where \( r = 2 \).

(b) Note when \( r = 2 \) the volume is \( V(2) = 4 \cdot 2^3 - 2^2 = 4 \cdot 8 - 4 = 28 \). If the volume is decreased from 28 to 26 then estimate the change in the radius.

(c) If we measure the radius and get 2 with an error of \( \pm .1 \) then estimate the error and relative error using 28 as the volume of the can.
4. (20 Points) The number of guppies in a pond is modeled by the rate equation

\[ G' = 0.002G(1000 - G) \]

where \( G \) is the number of guppies in the pond and \( G' \) is the number rate of change measured in guppies per day.

(a) If the pond has no guppies and then 200 guppies are released in it, then are there more or less guppies on the next day? (HINT: Is the rate \( G' \) positive or negative?)

Answer: \__________\ Why?

(b) If in the last question 2,000 guppies are released then are there more or less guppies the next day? Answer: \__________\

(c) If we count and find that at some time there are 600 guppies, then approximately how many guppies are there 24 hours latter? Do this approximation by taking two steps of size 12 hours.

Answer: \__________\

(d) What is the number of guppies if the population is stable in the sense that the number of guppies stays same form day to day?

Answer: \__________\
(15 Points) Measurements are made of the length $L$ (measured in cm) of a brass rod at different temperatures $T$ (measured in °C). Some of the information involved is given in the table at the right.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$L$</th>
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<tr>
<td>16</td>
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</tr>
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<td>189.14</td>
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<td>24</td>
<td>189.27</td>
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(a) Give an estimate for the rate of change of $L$ with respect to $T$ when $T = 18^\circ$C. Include that the units answer.

(b) What is the microscope equation at the point where $T = 18^\circ$C?

(c) Estimate the temperature at which the length is 188.83 cm.
6. (10 Points) Below is a graph of distance \( D(t) \) as a function of time \( t \). Then sketch a graph of velocity, which is the same as the derivative \( D'(t) \).