

# Test 1

Name: \_\_\_\_\_

**Show your work!** Answers that are not in the form of a sentence and do not have an adequate explication of why they work will receive no credit.

1. (20 points) A man starts running along a straight road at a constant speed starting from his house. At 2:30PM he is 4 miles from home and 15 minutes later he is 6 miles from home.

(a) How fast is he running?

(b) At what time did he leave home?

(c) Give a formula for his distance from home in terms of the time. (Be sure to label all of your variables.)

2. (20 points) Consider the following S-I-R model for the spread of chicken pox.

$$\begin{aligned}S' &= -.00002SI \\I' &= .00002SI - \frac{1}{8}I \\R' &= \frac{1}{8}I\end{aligned}$$

where  $S$  is the number of susceptibles,  $I$  is the number of infecteds, and  $R$  is the number of recovered, all measured in number of persons.  $S'$ ,  $I'$  and  $R'$  are the rates of change measured in number of persons per day.

(a) According to this model how long (on the average) is a person sick with the chicken pox?

Why?

(b) What is the threshold for  $S$ , so that if  $S$  is less than the threshold, the number on infecteds decreases, but if  $S$  is greater than the threshold the number of infecteds increases?

(c) Assume we find that on Monday morning the values of  $S$ ,  $I$  and  $R$  are  $S = 100,000$ ,  $I = 8,000$ , and  $R = 2,000$ . Then estimate the values of  $S$ ,  $I$  and  $R$  on Tuesday morning.

3. (15 points) At the beginning of the year 200 guppies (a type of small, but very fast breeding fish) are released in a pond. It is known that the *per capita* growth rate of guppies is 1.2 guppies per month.

(a) What is the rate equation for the growth of the population of guppies in the pond. Label all variables you use.

(b) Give an estimate for the number of guppies a month after the first two hundred were released.

(c) Use your answer to part (b) to give an estimate for the net rate of change of the guppy population a month after the first 200 were released and use this to estimate the number of guppies two months after the first 200 were released.

(d) Write a short paragraph explaining how you could be a more accurate estimate for the number of guppies two months after the first 200 were released.

4. (15 points) The area  $A$  of a square with side of length  $s$  is  $A = s^2$ .

(a) If  $s$  is increased from  $s = 2$  to  $s = 3$ , then what is the change in  $A$ ?

(b) If  $s$  is increased from  $s$  to  $s + \Delta s$  what is the corresponding change in  $A$ ?

(c) If  $s$  is increased by 30%, then what is the % increase in  $A$ ?

5. (10 points) The following is a graph of temperature  $T$ , in degrees F, as a function of time  $t$ , in hours over a three day period.

(a) What is the highest temperature during the three days?

(b) When did the lowest temperature occur?