Homework

Most of this assignment is just basic skills, that is basic drill on things like taking derivatives that should become automatic. As most of you will be away form computers for the break, this is set up so that you do not need anything other than pencil and paper. The basic rules for derivatives we have to date are:

Function	Derivative
$cf(x)$	$cf'(x)$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$
$f(g(x))$	$f'(g(x))g'(x)$

We also know the derivatives of the following functions:

Function	Derivative
cx^p	cpx^{p-1}
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\cot(x)$	$-\csc^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
b^x	$\ln(b)b^x$
e^x	e^x

Compute the derivatives of the following functions. If it is a function of more than one variable, then find all the partial derivatives.

$\frac{t^2-1}{\sqrt{5+t+3t^2}}$ $\quad ts+\sqrt{2t+st^2}$ $\quad \frac{y^3-1}{y-1}$

$y=\frac{x-\sqrt{x^2-1}}{2}$ $\quad \sqrt[3]{\frac{2-3x}{3-2x}}$ $\quad \frac{3}{(2x^2+5x)^{\frac{3}{2}}}$

$\frac{1-x^2}{1+x^2}$ $\quad x^2\sqrt{x^2-a^2}$ (with a constant) $\quad (x^2+y^2)\tan(xy^2z^3)$

$\frac{r(2-\cos(2\theta))}{r^2+z^2}$ $\quad \sec(u^2+3v)$ $\quad 5^{\sqrt{2-x+2y}}$

$x^2e^{2y+3z}\cos(4w)$ $\quad (s-3t)\cot(t)$ $\quad \csc(5\theta)$

$\tan\left(\frac{y}{x}\right)$ $\sec\left(\frac{y}{x}\right)$ $\csc\left(\frac{y}{x}\right)$

$e^{\sin(2x-y)}$ $\frac{1+\tan\theta}{1-\cot\theta}$ $e^{\frac{-x^2}{2\sqrt{t}}}$

$\tan(1+\cos(x+3y))$ $\frac{a-be^t}{a+be^t}$ with a, b constants $\sec^2(\theta)-\tan^2(\theta)$

$e^{x^2-1}\tan(4x)$ $\sin^3(x+e^{x/4})$ $\frac{x^2-x+2}{x-3}$

$\frac{(.7)^{3t}}{1+\sin t}$ $\cos\left(\frac{1-t}{1+t}\right)$ $\frac{2}{e^x+e^{-x}}$

1. Find the tangent line to graph of $y = x^3 - 2x + 4$ at the point where $x = -1$.
2. Let $f(x) = 2x^3 + 3x^2 - 12x + 5$.
   (a) Compute the derivative $f'(x)$ of $f(x)$ and factor it into linear factors.
   (b) Use your answer to that last part to find the intervals where the derivative $f'(x)$ is positive and the intervals where $f'(x)$ is negative.
   (c) On what intervals is the function $f(x)$ increasing? HINT: Recall that a function is increasing when its derivative is positive and decreasing when its derivative is negative.
   (d) Make a rough graph of $y = f(x)$ on the interval $-3 \le x \le -1$.
   (e) What is the largest value that $f(x)$ takes on in the interval $-3 \le x \le -1$? At what value of x does this maximum occur? What is the value of the derivative $f'(x)$ at this point? Explain why the derivative has this value at a maximum.
3. Let $f(x)$ be a function so that its derivative is proportional to its square. Write a differential equation for $f(x)$.
4. A snowball melts so that the rate of decrease of its volume is proportional to its surface area. Find the rate equation satisfied by the radius of the snowball.