

Test 3

Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (30 points) Find the derivatives of the following functions. If it is function of more than one variable, then find all the partial derivatives. **Circle your answers.**

(a) $-4x^7 + \sqrt{x}$.

(b) $e^\theta(\cos \theta + 2 \tan \theta)$

(c) $\exp\left(\frac{x}{y}\right)$

(d) $\frac{x^2 + 1}{x^3 - 1}$

(e) $x^3 e^{2y+3z}$

(f) $\cos(\sqrt{t^2 + a^2})$, (where a is a constant.)

2 (10 points) Show that $y = x^3 - 1$ is a solution to the initial value problem

$$\frac{1}{3}xy' = y + 1, \quad y(1) = 0.$$

3.(20 points) The per capita growth rate of the population of South Carolina is 5.8 persons per year per 1,000 persons. At the beginning of 1990 the population of South Carolina was 3,121,833.

(a) What is the initial value problem satisfied by population of South Carolina? Label all variables (both dependent and independent) and give the units of measurement used.

(b) What is the solution to this initial value problem?

(c) What does this model predict for the population of South Carolina in the year 2000?

4. (15 points) (a) Write the microscope equation for the function $f(x, y) = x^4 + yx$ at the point where $x = 1$, and $y = 2$.

(b) If f is increased by .5 and y is decreased by .2 then approximate the change in x .

5. (15 points) Let $y(t)$ be a function satisfying the initial value problem

$$y' = (t - 1)3^t, \quad y(0) = 1.$$

(a) On what interval is $y(t)$ an increasing function?

(b) Sketch a graph of $y(t)$.

(c) At what value of t does $y(t)$ have a minimum?

6. (10 points) When a cold spoon is used to stir a hot cup of coffee, the coffee cools down and the spoon heats up. By Newton's law of cooling the rate of changes of the temperatures of both the coffee and the spoon are proportional to the difference in their temperatures. Write a system of differential equations for the temperatures of the coffee and the spoon labeling all independent and dependent variables.