Test 1

Name: __________________________

Show your work! Answers that do not have a justification will receive no credit.

1. (25 points) Find the derivatives of the following:
   (a) \( f(x) = 7x^3 - 9x^2 + 3x - 4 \).

\[ f'(x) = \]

(b) \( V = 4s^2 - 3\sqrt{s^3} \)

\[ \frac{dV}{ds} = \]

(c) \( h(t) = \sqrt{4t^2 + 1} \)

\[ h'(t) = \]

(d) \( H(\theta) = \sin \theta + 2\cos \theta + 3\tan \theta \)

\[ H'(\theta) = \]

(e) \( D = 2 \cdot 4^\frac{1}{2} + \frac{7}{t^5} \)

\[ \frac{dD}{dt} = \]

(f) \( P(n) = P_0(1.09)^n \), (where \( P_0 \) is a constant.)

\[ P'(n) = \]

(g) \( A(\alpha) = 5\cos^3(\alpha) \)

\[ A'(\alpha) = \]
2. (10 points) Measurements of the temperature (in degrees F) of a cup of hot water are made every 10 seconds. Some of the measurements are given in the table. What (approximately) is the rate of the temperature when \( t = 100 \) secs?

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Temp (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>93.50</td>
</tr>
<tr>
<td>90</td>
<td>93.15</td>
</tr>
<tr>
<td>100</td>
<td>92.80</td>
</tr>
<tr>
<td>110</td>
<td>92.45</td>
</tr>
<tr>
<td>120</td>
<td>92.10</td>
</tr>
</tbody>
</table>

3. (10 points) Let \( V(s) = s^3 + s \). Write the microscope equation at the point where \( s = 2 \).

4. (15 points) Fill in the blanks.
(a) If \( f(4) = 5 \) and \( f'(4) = 6 \) a reasonable estimate of \( f(4.2) \) is ___.

(b) If \( g(5) = 6 \) and \( g'(5) = .4 \) a reasonable estimate of \( g(4.5) \) is ___.

(c) If \( h(3) = .5 \) and \( h'(3) = 2 \) a reasonable estimate of \( h(_______) \) is 0.
5. (20 points) Let \( y = f(x) \) have the graph as shown. Then answer the following.

(a) What is \( f'(1.9) \)? \\

(b) For what values of \( x \) is \( f'(x) = 0 \)? \\

(c) On what intervals is \( f'(x) \) negative? \\

(d) Draw your own axis and sketch a graph of the derivative \( y = f'(x) \).
6. (20 points) A snow ball is brought into a warm room. Let $V(t)$ be the volume of the snow ball (measured in cubic inches) after $t$ minutes after it was brought into the room. It is known that the volume satisfies the rate equation

$$V'(t) = -\frac{1}{3} V(t)^{\frac{2}{3}}.$$ 

(a) Five minutes after the snow ball was brought into the room its volume is 8in$^3$. Write the microscope equation relating $\Delta V$ and $\Delta t$ at the point where $t = 5$.

(b) Using the data from part (a) estimate the volume the snowball when $t = 5.3$.

(c) Again using the data from part (a) estimate the time when the volume was 9in$^3$. 