Review for Test 2

As usual you are still responsible for the maternal on the first two exams.

1. **Computing derivatives and partial derivatives.** At least 25% of the test will be on this. Most of these questions will not be tricky, but only involve knowing the derivatives of the basic functions and rules. Being able to do the problems of the homework involving derivatives will be more than enough. Note that we have several different notations for the partial derivatives. For example

$$\frac{\partial f}{\partial x} = f_x = \frac{\partial}{\partial x} f.$$

2. **The microscope equation.** We now have this for functions of bout one and several variables. In one variable, if $y = f(x)$, then the microscope equation at $x = a$ is $\Delta y \approx f'(a)\Delta x$. If $z = f(x, y)$ is a function of two variables and then the microscope equation at $x = a, y = b$ is

$$\Delta z \approx \frac{\partial f}{\partial x})(a, b)\Delta x + \frac{\partial f}{\partial y}\Delta y.$$

with obvious variants when there are three or more variables. Besides just writing the microscope equation (which after all involves nothing more than taking the derivatives and then plugging the appropriate values) you should expect a word problem involving the microscope equations.
Sample problem. The volume of a pyramid with square base of side length s and of height h is $V(h, s) = \frac{1}{6}hs^2$. Thus when $h = 2$ and $s = 3$ the volume is $V(2, 3) = \frac{1}{6} \cdot 2 \cdot 3^2$. If the volume is increased form 3 to 3.1 and the height is increased from 2 to 2.2, then what approximates change in the length of the side of the base? Is the new base longer or shorter?
Step 1 of solution. Write the microscope equation for V at $h = 2$ and $s = 3$. This is $\Delta V \approx \frac{3}{2}\Delta h + 2\Delta s$.
Step 2 of solution. Use $\Delta V = 3.1 - 3 = .1$ and $\Delta h = 2.2 - 2$ in the microscope equation and solve for Δs to get $\Delta s \approx -.1$. This is the approximates change in the side length. As this is negative the new side is smaller.

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3. **Rate equations** *also known as* **Differential equations.**

   (a) Know what it means for a function, or functions, to be a solution to a system of differential equations. Also know what it means to be a solution to an initial value problem. As a sample problem: Are $u(t) = \cos(t)$ and $v(t) = \sin(t)$ a solution to the initial value problem

$$\begin{aligned} u'(t) &= v(t), & u(0) = 1 \\ v'(t) &= -u(t), & v(0) = 0 \end{aligned}$$

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   (b) **The exponential function and the population growth equation.** For any constant k we have that

$$y(t) = Ce^{kt}, \quad \text{is the unique solution to} \quad y' = ky, \quad y(0) = C.$$

The equation $y = ky$ is the equation for population growth. **Important note:** we also use the notation $\exp(t) = e^t$. For practice in the use of this notation do problem 5 on Page 235 You should read §4.3 pages 233–241. Practice: Pages 234–235 3,4 Page 136 8,9.

(c) There will be at least one problem on setting up a rate equation. In doing this it is very likely that you will have to understand what the word proportional means.