Review for the Final

This hopefully covers the highlights of what we have covered this term and should be good practice for the final.

1. **Functions.** You will be expected to have a reasonable idea of what a function is and its relationship to its graph. At the beginning of the term we did a good number of problems where you had to graph function form a verbal description. You should expect some problems of this nature. As review look at the problems on the first work sheet (the one with the jogger problem). You should expect problems where you have to write a formula for a function given a verbal description of it (for example like problem 3 on test 1). Also given a graph of a function you should be able to do things like tell on what intervals it is increasing or decreasing, where the local maxima and minima are located etc.

2. **Derivatives and Local Linearity.** This is of course the high light of the class. As in the last test there will be a fair number of problems involving just computing derivatives of functions. The only new function we have encountered since the last test is the logarithm. The logarithm to the base b where $b > 0$ and $b \neq 1$ is defined by either of the equations

$$\log_b(b^x) = x, \qquad b^{\log_b(x)} = x.$$

The most important base for logarithms is the number $e = 2.7182818\ldots$ Because of this we use the shortened notation $\ln(x)$ for $\log_e(x)$. What makes e the best base for logarithms (at least form the point of view of calculus) is the formula

$$\frac{d}{dx}\ln(x) = \frac{1}{x}.$$

Note that we have the change of base formula

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$

so that logarithms to any base can be expressed in terms of ln which is important for taking derivatives of these functions.

Approximating the derivative. Be able to give a good approximation to the derivative for a function given by a table or a graph. For example on test 2 look at problems 2 and 5.

The microscope equation. This is the equation that expresses the fact that a smooth function is "locally linear". You should be sure that you know the microscope equation equation both for functions of one variable and for functions of several variables. (Along the same lines you should be able to find the tangent line to the graph of a function of one variable.) You should also be able to use the microscope equations to do approximations. For example look at Problem 4 on test 2, problem 4 on test 3. Problem 7 page 172 of the text.

3. **Rate equations.** There will be problems on setting up rate equations and also problems on using them to be information about solutions. You need to know what it means for a function to be a solution to a differential equation (which is just anther name for a rate equation) and what it means to be a solution to an initial value problem. Here you show look at test 1 problems 4 and 5, test 2 problem 6, test 3 problems 2 and 6. There will also be problems of the type given $y' = f(t)$, find y. Here you should look at problem 1 page 263.

The one initial value problem we know the explicit solution to is: *The only solution to* $y' = ky$, $y(0) = C$ *is* $y(t) = Ce^{kt}$. (That is the population growth equation.) It is likely there will be problems related to this equation. A good practice problem is page 252 problem 2 of the text and problem 3 on test 3.

Approximating solutions to rate equations. This is nothing more that combining a rate equation withe the microscope equation to give an approximation. Examples of this are test 1 problem 4 part (c), the quiz on the growth of kudzu, test 2 problem 6 parts (b) and (c). To improve the accuracy of an approximation you just take a larger number of small steps (recall this is *Euler's method*).

Even when we can not solve a rate equation implicitly it is possible to use it to get information about solutions. Examples this are test 1 problem 6 parts (a), (b) and (c) and test 3 problem 5 and the graph problems we have been doing in class the last week or so.

4. **Various and Sundry Surprise Mystery questions.** These will involve graphing functions, finding maxima and minima, and the meaning of life.