Final

Name:______________________________

Show your work! Answers that do not have a justification will receive no credit.
Circle your answers.

1. (40 points) Find the derivatives of the following functions. If it is function of more than one variable, then find all the partial derivatives. You do not have to simplify your answers.

(a) $9x^3 + \sqrt[3]{x+1}$.

(b) $\dfrac{1-\cos(x)}{1+\cos(x)}$

(c) $\ln(x^4 + 3x)$

(d) $\dfrac{\log_5(xy^2)}{y}$

(e) $e^{\tan(2\theta)}$

(f) $\cos(\theta^2 + 1)\sec(\theta^2 + 1)$

(g) $\sqrt{x + e^y}$

(h) 13^{4u-9}.

(i) $u^4 \cot(v+2w)$.

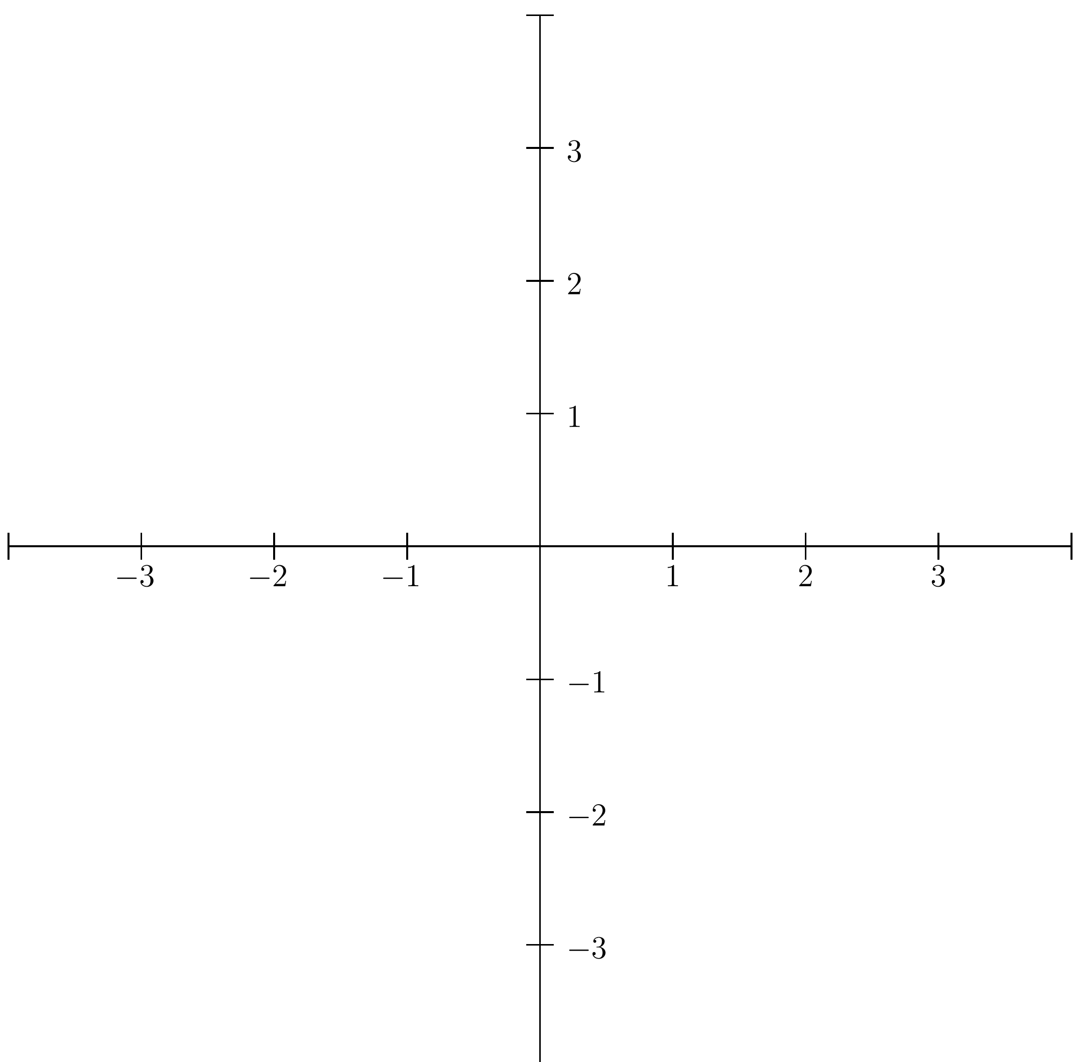
(j) $\sqrt{\ln(t^2+5)}$.

2 (20 points) (a) Write the microscope equation for $y = 4x^2 - 2x$ at the point where $x = -3$.

(b) What is the equation of the tangent line to $y = 4x^2 - 2x$ at the point where $x = -3$?

(c) Write the full microscope equation for $V = \sqrt{u^2 + v^2}$ at the point where $(u, v) = (3, 4)$.

3.(15 points) Sketch the graph the solution to the initial value problem $y' = \dfrac{4 - t^2}{1 + y^2}$, $y(2) = 0$ and label all the local maxima and minima.



4. (20 points) A truck driver starts a trip by driving at 50mph for 2 hours and then speeds up to 60 mph for 3 hours. She then stops for lunch which takes an hour and resumes the trip by driving at 40 mph for anther 5 hours.

(a) Graph the speed of the truck as a function of the time t in hours form the start of the trip. (draw and label your own axis.)

(b) Find formulas for the distance $D(t)$ covered for t in the intervals (i) $2 < t < 5$ and (ii) $6 < t < 11$.

(c) How long (starting from the beginning of the trip) does it take for the truck to cover 400 miles?

T	L
56	188.79
58	188.90
60	189.01
62	189.12
64	189.23

5. (15 points) Measurements are made of the length L (measured in cm) of a brass rod at different temperatures . Some the information involved is given in the table at the right (measured in °F.

(a) Give an estimate for the rate of change of L with respect to T.

(b) Write the microscope equation at the point where $T = 60°\text{F}$.

(c) Estimate the temperature at which the length is 188.95

6. (20 points) At the beginning of the year 200 guppies are released into Lake Murray. For the first several years the rate of change of the numbers of guppies in the lake is proportional to the number of guppies in the lake. After 2 years there are 1500 guppies in the lake.

(a) Give a formula for the number of guppies after t years.

(b) How may years does it take before there are a 1,000,000 guppies in the lake?

7. (15 points) Fill in the blanks.

(a) If $f(3,-2) = 4$, $\frac{\partial f}{\partial x}(3,-2) = 2$ and $\frac{\partial f}{\partial y}(3,-2) = -3$, then a reasonable estimate of

$f(3.2,-2.1)$ is __________.

(b) If $g(2,3) = 1$, $\frac{\partial g}{\partial x}(2,3) = 2$, $\frac{\partial g}{\partial y}(2,3) = -3$, then a reasonable estimate of the solution

to $g(1.8, y) = 0$ is __________.

(c) If $h(3,2) = 1$, $h(3.1, 1.9) = 1.3$ and $\frac{\partial h}{\partial y}(3,2) = -2$, then a reasonable estimate of

$\frac{\partial h}{\partial x}(3,2)$ is __________.

8. (20 points) Solve the following initial value problems.

(a) $A'(t) = 0$, $A(3) = 7$.

(b) $y'(t) = -5y(t)$, $y(0) - 14$.

(c) $y'(t) = 4t^3 + 6t^2 - 6t + 1$, $y(0) = 4$.

(d) $y'(t) = te^{t^2}$, $y(0) = -1$.

9. (20 points) Of all rectangles with perimeter of length 40, which one has the largest area?

10. (20 points) (a) Show that $x(t) = \cos(t)$, $y(t) = \sin(t)$ is a solution to the initial value problem

$$\begin{aligned} x'(t) &= -y(t), \qquad & x(0) &= 1 \\ y'(0) &= x(t), \qquad & y(0) &= 0. \end{aligned}$$

(b) Find the value of the constant a so that $u(t) = 4e^{-3t} + a$ is a solution to

$$u' = -3(u - 70).$$