## Mathematics 122 Test \#2

You are to use your own calculator, no sharing.
Show your work to get credit.
A blank page is attached for scratch work.
(1) (45 points) Compute the following derivatives. Assume that $a, b$, and $c$ are constants.
(a) $p=5 t^{6}-3 t^{4}+$
(b) $C=\frac{3}{\sqrt{q}}-\frac{5}{5 q^{4}}$ $\frac{d p}{d t}=$ $\qquad$
(c) $y=-3 e^{x}+7 \ln (x)$
$y^{\prime}=$ $\qquad$
(d) $R=3 b \cdot 7^{t}$
(e) $y=-5(4 x+2)^{13}$
$y^{\prime}=$ $\qquad$
(f) $R(x)=8 e^{3 x^{2}}$

$$
R^{\prime}(x)=
$$

$\qquad$
(g) $w=5 \ln \left(z^{3}+z^{2}\right)$

$$
\frac{d w}{d z}=
$$

$\qquad$
(h) $A(r)=4 \sqrt{e^{r}+r}$

$$
A^{\prime}(r)=
$$

$\qquad$
(i) $F(t)=5 e^{3 t^{4}}$

$$
F^{\prime}(t)=
$$

$\qquad$
(j) $y=\sqrt{x} e^{3 x^{2}}$

$$
y^{\prime}=
$$

$\qquad$
(k) $u=\frac{t^{2}-1}{t^{2}+1}$

$$
\frac{d u}{d t}=
$$

$\qquad$
(l) $H(r)=(3 r+1)\left(r^{4}+r^{2}\right)^{5}$
$H^{\prime}(r)=$ $\qquad$
(m) $p=2 \cos (\theta)+3 \sin (\theta)$ $\frac{d p}{d \theta}=$ $\qquad$
(n) $y=\cos (2 x)+\sin (3 x)$

$$
y^{\prime}=
$$

$\qquad$
(o) $A(r)=3 \cos ^{4}(6 r)$

$$
A^{\prime}(r)=
$$

$\qquad$
(2) (10 Points) Find the second derivatives of the following functions.
(a) $y=7 x^{3}+2 x^{2}-9 x+11$
$y^{\prime \prime}=$
(b) $w=3 e^{2 z}+4 \ln (z)$

$$
\frac{d^{2} w}{d z^{2}}=
$$

$\qquad$
$\qquad$
(3) (5 points) Find the tangent line to $y=3 x^{3}-5 x$ at the point where $x=2$.
(4) (15 Points) Draw graphs of functions with the following properties
(a) $f$ is increasing at a decreasing rate.
(b) $f^{\prime}<0$ and $f^{\prime \prime}>0$.
(c) $f(3)=1, f^{\prime}(3)=0$ and $f^{\prime \prime}(x)<0$
(d) - $f(1)=-3, f(2)=1, f(4)=-2$,

- $f^{\prime}(1)=f^{\prime}(2)=f^{\prime}(4)=0$,
- $f^{\prime}(x)<0$ for $x<1$ and $2<x<4$,
- $f^{\prime}(x)>0$ for $1<x<2$ and $4<x$.
(5) (5 Points) In the figure

(a) At which of the labeled points is $f(x)>0$.
(b) At which of the labeled points is $f^{\prime}(x)>0$.
(c) At which of the labeled points is $f^{\prime \prime}(x)>0$.

(6) (7 Points) For a function given by the table | $t$ | 2.0 | 2.2 | 2.4 | 2.6 |
| :---: | :--- | :--- | :--- | :--- | :--- |
| $A(t)$ | 3.7 | 3.4 | 2.9 | 2.0 |

(a) Make a table for $A^{\prime}(t)$.
(b) Make a table for $A^{\prime \prime}(t)$.
(7) (8 Points) A roast chicken is taken out of the oven to cool. Let $T(t)$ be temperature of the chicken in degrees Fahrenheit $t$ minutes after it was removed from the oven. Assume that $T(20)=120$ and that $T^{\prime}(20)=-3.5$
(a) Explain why $T^{\prime}(20)$ is negative.
(b) What are the units on $T(20)$ ?
(c) What are the units on $T^{\prime}(20)$ ?
(d) Estimate $T(22)$.
(e) Estimate $T(19)$.

$$
T(22) \approx
$$

$\qquad$
$T(19) \approx$ $\qquad$
(8) (10 Points) A group of students decide to sell Gamecock coffee cups. Figure 1 shows the cost $C(q)$ (in dollars) and revenue $R(q)$ (in dollars) for selling $q$ of the head bands.


Figure 1
(a) What are the startup costs for the students?
(b) At what price are the students selling the coffee cups? $\qquad$
(c) Estimate the cost of producing the 125 th coffee. (That is estimate the marginal cost $C^{\prime}(125)$.

$$
C^{\prime}(125) \approx
$$

$\qquad$
(d) Estimate the number of coffee cups the students should sell to maximze their profit.

