Mathematics 122 Test #2

You are to use your own calculator, no sharing.

Show your work to get credit. This means that if you use your calculator to solve a problem, then you have to write a sentence telling how you used it to do the calculations. (That is if you graphed it and zoomed in then say that is what you did etc.)

1. (15 points) Draw Graphs of functions that satisfy the following:
   (a) Is increasing at an decreasing rate.
   (b) Is decreasing and concave up.
   (c) Has $f'(x) < 0$ and $f''(x) < 0$
   (d) has $f''(x) < 0$ and $f'(1) = 0$. 
2. (20 Points) A group of students decide to market a guide to the bars of Five Points. The following graph shows the cost, $C(q)$, and the revenue, $R(q)$, from producing a quantity $q$ of the guides.

(a) About how much were the startup costs of the for producing the guides? 

(b) From the graph roughly how much is the marginal revenue $R'(100)$ of producing a 100 copies of the guide? 

(c) If the students are producing 275 guides then is it in their interests to produce more booklets? Write a sentence or two explaining your answer. (Having the explanation is the most important part of the answer.) 

(d) Make a guess at the number of guides they should product to maximize their profit.
3. (15 points) For the functions given by the following graphs sketch a graph of the derivative on the same set of axis.
4. (10 points) Sketch the graph of a function $y = f(x)$ so that
- $f(1) = 3,$
- $f(4) = 7,$
- $f(6) = -1,$
- $f'(x) < 0$ for $x < 1,$
- $f'(x) > 0$ for $1 < x < 4,$
- $f'(x) < 0$ for $4 < x < 6,$ and
- $f'(x) > 0$ for $6 < x.$

Your graph should not have any sharp corners.
5. (15 points) Let a function \( w(t) \) be given by the table

<table>
<thead>
<tr>
<th>( t )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w(t) )</td>
<td>13.46</td>
<td>9.03</td>
<td>6.05</td>
<td>4.06</td>
</tr>
</tbody>
</table>

(a) Approximate the derivative at the points 3, 5, 7.

<table>
<thead>
<tr>
<th>( t )</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w'(t) )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Is the second derivative \( w''(t) \) positive or negative? Explain your answer.

(c) Estimate \( w''(4) \) \( w''(4) \approx \)
6. (15 Points) A car goes 50mph for 45min. It then goes for 60mph for a hour and slows down to 40mph for a hour and 15min.
(a) Sketch a graph of the speed of the care as a function of time

(b) How far did the car travel in the first two hours of the trip? 

(c) Sketch a graph of distance traveled as a function of time.
7. (5 Points) Let \( f(x) = \frac{1}{x} \). Then give an upper bound on \( \int_{1}^{3} f(x) \, dx \) by splitting the interval \([1, 3]\) into four equal length subintervals and computing the upper sum.

Upper bound = 

8. (5 Points) Use your calculator to compute \( \int_{1}^{3} \frac{2^x}{1 + x} \, dx \). 
